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# THE ASTROPHYSICAL JOURNAL

## AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

*Edited by*

GEORGE E. HALE
Mount Wilson Observatory of the Carnegie Institution of Washington

EDWIN B. FROST
Yerkes Observatory of the University of Chicago

HENRY G. GALE
Ryerson Physical Laboratory of the University of Chicago

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## An International Review of Spectroscopy and Astronomical Physics


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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME LXVIII JULY 1928 NUMBER 1

# A CALIBRATION OF ROWLAND'S SCALE OF INTENSITIES FOR SOLAR LINES[1]

BY HENRY NORRIS RUSSELL,[2] WALTER S. ADAMS, AND CHARLOTTE E. MOORE

ABSTRACT

*Calibration of Rowland intensities.*—The relative intensities in multiplets are expressed by formulae, based on the correspondence principle, which involve only the quantum numbers of the lines. Although approximate, these formulae can probably be depended on to give correct average results. Since these theoretical intensities may be assumed to be proportional to the numbers of active atoms producing the lines, the way is open to a calibration of Rowland's scale in terms of $N$ (number of active atoms). Data on 228 multiplets gave a provisional relation between log $N$ and $R$ which was found to depend on wave-length. Correction by successive approximations gave finally

$$\log N = B \log A ,$$

where $A$ (Table III) is a function of $R$, and $B$ (Table IV) a function of $\lambda$. The formula is generally applicable to all the elements studied, but uncertainty in the zero point restricts its use to neighboring lines; even for such lines it gives only relative values of $N$. The outstanding discordances would be wholly accounted for by a probable error in Rowland's intensities of $\pm 0.64$ units of his own scale.

Rowland's estimates of the intensities of the Fraunhofer lines, though obviously rather rough, afford a valuable storehouse of information. In order to utilize this, the meaning of his arbitrary scale must be determined. This may be attempted in two ways.

First, the contours of an absorption line may be observed and the amount of light which it subtracts from the solar emission may be determined. Measures of this sort have recently been made by

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 358.

[2] Research Associate of the Mount Wilson Observatory, Carnegie Institution of Washington.

von Klüber[1] and by Minnaert.[2] The latter finds, for example, that the loss of light in $H\alpha$ corresponds to the complete extinction of a strip of spectrum 4.2 A wide, and $\lambda$ 6634 (Rowland intensity 5) to one 0.23 A in width. The value and importance of such work are obvious. But, even when the laborious measures which it will require have been completed, the physical interpretation of the results will not be easy. It is by no means safe to assume that if one line blocks out ten times as much solar energy as another, ten times more atoms conspire to produce the first than the second. Indeed, a satisfactory theory of the widths and depths of the Fraunhofer lines is very difficult. It is more likely to come as the final product of a general theory of the solar atmosphere than as a step toward its development.

The other type of calibration is one which attempts to determine directly the relative numbers of atoms in the solar atmosphere which are effective in producing lines of various intensities. If this proves possible, the long and difficult theoretical investigations just mentioned are circumvented, and the observed intensities become available at once for many astrophysical purposes. Moreover, the data which are obtained should prove of importance in testing theories of the formation and contour of the lines. It is rarely that a royal road of this sort can be found, but in the present case one really appears to be open.

Much work has been done in recent years upon the intensities of lines in multiplets, culminating in the derivation, independently and almost simultaneously by other investigators,[3] of formulae for the relative intensities, based upon the correspondence principle and depending only on the quantum numbers of the lines involved. Measures show that these formulae are only approximate.[4] In many cases, especially in wide multiplets, the deviations are considerable, but in others the agreement is very good. It appears safe to assume that the formulae may be trusted on the average—individual deviations tending to vanish in the mean.

[1] *Zeitschrift für Physik*, **45**, 610, 1927. [2] *Ibid.*, **44**, 481, 1927.

[3] R. de L. Kronig, *ibid.*, **31**, 885, 1925; A. Sommerfeld and H. Hönl, *Sitzungsberichte der Preussischen Akademie der Wissenschaften*, p. 141, 1925; H. N. Russell, *Proceedings of the National Academy of Sciences*, **11**, 314, 1925.

[4] Cf. L. S. Ornstein, *Physikalische Zeitschrift*, **28**, 693, 1927.

According to Ornstein,[1] the quantities given by the formulae represent approximately the relative numbers of "fictitious resonators," all of equal amplitude, which are involved in the production of the lines. To find the energy radiated, they must be multiplied by factors proportional to $\nu^4$, $\nu$ being the frequency. To treat the numbers of these hypothetical resonators as proportional to the numbers of atoms which are acting is sufficient for our purpose, especially as no comparison between lines in widely separated parts of the spectrum can yet be made.

In the course of preparation of the Revision of Rowland's Table, all the lines which belong to known multiplets have been recorded, so that extensive material was available. How it was treated is best illustrated by an example: There is a multiplet of *Fe* I, of type $^5D'-{}^5F'$, comprising twelve lines, of which one is blended in the sun. Details for these lines are given in Table I, in which $R$ is Rowland's intensity and $N$ denotes the relative numbers of atoms which should theoretically be active in producing the lines, on the basis that the number for the strongest line is unity. Rejecting the blended line and taking means for the three groups of various theoretical intensities, we find:

TABLE I

| λ | Designation | log $N$ | $R$ | λ | Designation | log $N$ | $R$ |
|---|---|---|---|---|---|---|---|
| 4736..... | $^5D_4-{}^5F_5$ | 0.00 | 6 | 4625..... | $^5D_3-{}^5F_3$ | −0.72 | 5 |
| 4707..... | $^5D_3-{}^5F_4$ | − .17 | 5 | 4607..... | $^5D_2-{}^5F_2$ | .74 | 4 |
| 4668..... | $^5D_2-{}^5F_3$ | .36 | 4 | 4598..... | $^5D_1-{}^5F_1$ | 0.90 | 3 |
| 4637..... | $^5D_1-{}^5F_2$ | .59 | 5 | | | | |
| | | | | 4574..... | $^5D_4-{}^5F_3$ | 2.04 | 1 |
| 4613..... | $^5D_0-{}^5F_1$ | .90 | 3 | 4568..... | $^5D_2-{}^5F_1$ | 1.74 | 1 |
| 4654..... | $^5D_4-{}^5F_4$ | −0.86 | 5 | (4565..... | $^5D_3-{}^5F_2$ | −1.74 | 2)* |

* Blend with *Co*.

| Lines | log $N$ | $R$ | Δ log $N$ | Δ$R$ | Mean $R$ |
|---|---|---|---|---|---|
| 4..... | −0.28 | 5.0 | | | |
| | | | 0.54 | 1.0 | 4.5 |
| 5..... | 0.82 | 4.0 | | | |
| | | | 1.07 | 3.0 | 2.5 |
| 2..... | −1.89 | 1.0 | | | |

[1] *Zeitschrift für Physik*, **40**, 412, 1926.

Each multiplet, treated in similar fashion, gives one or two (rarely three) values of the increment $\Delta R$ corresponding to a theoretical increment $\Delta \log N$. A summary of the data which were employed is given in Table II. This shows, for each element, the number of multiplets, of unblended lines available, and of the resulting pairs of values $\Delta R$ and $\Delta \log N$.

TABLE II

NUMBERS OF MULTIPLETS, LINES, AND INCREMENTS

| Element | Multiplets | Lines | $\Delta R$ and $\Delta \log N$ | Element | Multiplets | Lines | $\Delta R$ and $\Delta \log N$ |
|---|---|---|---|---|---|---|---|
| *Fe* | 46 | 340 | 75 | *Mg* | 2 | 6 | 4 |
| *Ti* | 41 | 224 | 52 | *Al* | 1 | 2 | 1 |
| *Cr* | 31 | 181 | 41 | *Na* | 1 | 2 | 1 |
| *Ni* | 18 | 88 | 23 | *Fe+* | 6 | 41 | 11 |
| *V* | 18 | 103 | 26 | *Ti+* | 26 | 107 | 28 |
| *Mn* | 16 | 89 | 20 | *Cr+* | 7 | 35 | 9 |
| *Ca* | 8 | 40 | 13 | *Sc+* | 7 | 30 | 8 |
| Total | | | | | 228 | 1288 | 312 |

Only the strongest lines of *Mg*, *Al*, and *Na* were included. The numerous intersystem combinations could not be used, as there is no satisfactory theory of intensities in such groups. The available material is, however, extensive enough.

The cases in which lines of about the same mean Rowland intensity were compared were then grouped, mean values of $\Delta R$ and $\Delta \log N$ being taken to diminish the effect of the obviously large accidental errors of Rowland's estimates. A curve was then drawn, giving $\Delta \log N/\Delta R$ as a function of $R$, and from this $\log N$ itself was found. With the aid of this preliminary calibration curve, the values of $\Delta \log N$, corresponding to the observed difference in the Rowland intensities, were found for 306 of the line-groups listed in Table II. Comparison of these with the theoretical differences showed that the gradation scale of Rowland's estimates varies with the wave-length. A given increase in $\log N$ produces much more change in his numbers in the red than in the ultra-violet.

To allow for this it was assumed that

$$\log N = B \log A,$$

where $A$ depends on $R$ and $B$ on $\lambda$. The values of these functions were derived by successive approximations and are given in Tables

III and IV. The intensities $-1$, $-2$, and $-3$ have been substituted for Rowland's oo, ooo, and oooo, as has been done in the Revision.

It is evident that Rowland's scale cannot be represented by any simple function of the number of atoms involved. The empirical function log $A$ is well defined by the data, except at the upper end. There are only twenty-eight solar lines for which Rowland's intensities exceed 15, and but two exceeding 40. For this group the

TABLE III

| $R$ | log $A$ | $R$ | log $A$ | $R$ | log $A$ |
|---|---|---|---|---|---|
| −3 | −1.80 | 6 | 2.68 | 15 | 3.54 |
| −2 | −1.15 | 7 | 2.87 | 16 | 3.58 |
| −1 | −0.54 | 8 | 3.00 | 18 | 3.66 |
| 0 | .00 | 9 | 3.11 | 20 | 3.72 |
| 1 | +0.52 | 10 | 3.21 | 25 | 3.87 |
| 2 | +1.03 | 11 | 3.30 | 30 | 3.98 |
| 3 | +1.52 | 12 | 3.37 | 35 | 4.08 |
| 4 | +2.00 | 13 | 3.43 | 40 | 4.17 |
| 5 | +2.39 | 14 | 3.49 | | |

TABLE IV

| λ | $B$ | λ | $B$ | λ | $B$ | λ | $B$ |
|---|---|---|---|---|---|---|---|
| 3000 | 1.41 | 4000 | 1.15 | 5000 | 1.00 | 6000 | 0.89 |
| 3100 | 1.38 | 4100 | 1.13 | 5100 | 0.99 | 6100 | .88 |
| 3200 | 1.35 | 4200 | 1.11 | 5200 | .97 | 6200 | .87 |
| 3300 | 1.32 | 4300 | 1.10 | 5300 | .96 | 6300 | .86 |
| 3400 | 1.29 | 4400 | 1.08 | 5400 | .95 | 6400 | .85 |
| 3500 | 1.25 | 4500 | 1.06 | 5500 | .94 | 6500 | .85 |
| 3600 | 1.23 | 4600 | 1.05 | 5600 | .93 | 6600 | .84 |
| 3700 | 1.20 | 4700 | 1.04 | 5700 | .92 | 6700 | .83 |
| 3800 | 1.18 | 4800 | 1.02 | 5800 | .91 | 6800 | .83 |
| 3900 | 1.16 | 4900 | 1.01 | 5900 | 0.90 | 6900 | 0.82 |

tabular data are based on the assumption that $\Delta \log A = 1.5\, \Delta \log R$, which is the mean value indicated by the few multiplets available.

The correcting factor $B$ is well determined over its whole range, and suffices to reduce the *scale* of intensities in different regions of the spectrum to a homogeneous system. There is, however, no present evidence as to what extent the zero point changes with the wave-length, i.e., whether a line in the red demands more or fewer atoms for its production than one of the same tabular intensity in the ultra-violet. Still less is there anything in the present data to indicate, even approximately, the absolute number of atoms (per

square centimeter in a column extending through the whole atmosphere) which produce a line of given intensity. Only the *relative* intensities of neighboring lines are here considered.

TABLE V

GROUPING BY INTENSITY

| Limits of Intensity | No. | Mean $R$ | Theoretical $\Delta \log N$ | Calibrated $\Delta \log N$ | Ratio |
|---|---|---|---|---|---|
| >15 | 8 | 20.5 | 0.43 | 0.47 | 1.10 |
| 8 to 15 | 13 | 10.7 | .58 | .60 | 1.03 |
| 6 to 8 | 16 | 6.4 | .75 | .89 | 1.19 |
| 5 to 6 | 15 | 5.4 | .67 | .58 | 0.85 |
| 4 to 5 | 19 | 4.4 | .75 | .73 | 0.97 |
| 3 to 4 | 38 | 3.4 | .76 | .76 | 1.00 |
| 2 to 3 | 48 | 2.4 | .74 | 0.67 | 0.90 |
| 1 to 2 | 44 | 1.4 | .92 | 1.02 | 1.10 |
| 0 to 1 | 44 | 0.6 | .87 | 0.99 | 1.14 |
| −1 to 0 | 33 | −0.5 | .80 | .81 | 1.01 |
| −2 to −1 | 25 | −1.4 | .83 | .70 | 0.84 |
| −3 to −2 | 9 | −2.3 | 0.65 | 0.56 | 0.86 |

TABLE VI

GROUPING BY WAVE-LENGTH

| λ | No. | Theoretical $\Delta \log N$ | Calibrated $\Delta \log N$ | Ratio |
|---|---|---|---|---|
| 2992–3400 | 47 | 0.83 | 0.75 | 0.90 |
| 3400–3800 | 39 | .76 | .82 | 1.08 |
| 3800–4200 | 33 | .74 | .72 | 0.97 |
| 4200–4400 | 31 | .69 | .64 | 0.94 |
| 4400–4600 | 30 | .83 | .85 | 1.02 |
| 4600–5000 | 33 | .79 | .95 | 1.20 |
| 5000–5500 | 39 | .74 | .76 | 1.03 |
| 5500–6100 | 27 | .82 | .80 | 0.98 |
| 6100–6800 | 33 | 0.82 | 0.83 | 1.02 |

To test the validity of the calibration the values of log $N$ were computed with the aid of Tables III and IV for all the lines mentioned in Table II, and the values of $\Delta \log N$ derived from theory

and from the observations as here calibrated were compared in various groupings, with results illustrated in Tables V, VI, and VII.

The agreement of the theoretical values and those derived from the calibration in the case of Tables V and VI is merely an indication that the functions given in Tables III and IV need little, if any, systematic improvement. The agreement of the results for different elements, in Table VII, is, however, quite unforced, and

TABLE VII

GROUPING BY ELEMENTS

| Element | No. | Theoretical $\Delta \log N$ | Calibrated $\Delta \log N$ | Ratio |
|---|---|---|---|---|
| *Fe* | 75 | 0.80 | 0.79 | 0.99 |
| *Ti* | 52 | .98 | 1.02 | 1.04 |
| *Cr* | 41 | .53 | 0.53 | 1.00 |
| *Ni* | 23 | .96 | 1.09 | 1.14 |
| *V* | 26 | .72 | 0.68 | 0.95 |
| *Mn* | 20 | .60 | .64 | 1.06 |
| *Ca* | 13 | .85 | .79 | 0.93 |
| *Mg* | 4 | .35 | .30 | 0.86 |
| *Al* | 1 | .30 | .21 | 0.7 |
| *Na* | 1 | .30 | .24 | 0.8 |
| *Fe+* | 11 | .83 | .69 | 0.83 |
| *Ti+* | 28 | .85 | .90 | 1.07 |
| *Cr+* | 9 | .65 | .73 | 1.13 |
| *Sc+* | 8 | 0.83 | 0.76 | 0.92 |

indicates that the calibration is generally applicable. The outstanding discordances, which average $\pm 8$ per cent of the theoretical values (or $\pm 0.066$ in $\log N$),[1] are to be attributed mainly to the raggedness of Rowland's tabular estimates, which is obvious on inspection of the photographs. On the average, each of the groups in these tables is derived from thirty differences of intensity in multiplets between groups averaging 2.1 lines in number. The percentage discordance between calibration and theory for the value of $\log N$ derived from a single line should therefore be $0.0664 \times \sqrt{30} \times \sqrt{\frac{2}{2.1}}$,

[1] After excluding the two cases in which but a single pair of lines belong to an element.

or $\pm 0.37$. The corresponding probable error in $\log N$ is $\pm 0.32$, so that, for a single line, the estimate is as likely as not to be in error by a factor of 2. For the lines of intensity between $-2$ and 5, which form a large majority of those here studied, $\log N$ changes, on the average, by 0.50 per unit of $R$; hence a probable error in Rowland's estimates of $\pm 0.64$ unit of his own scale will account for the whole discrepancy.

The most obvious result of the present investigation is to emphasize the enormous differences in the numbers of atoms which are involved in the formation of the stronger and weaker Fraunhofer lines. From the weakest perceptible lines, of Rowland intensity $-3$ (or 0000), to such lines as *Hα* or the great iron lines in the violet (λλ 3720, 3735), this number increases by a factor of approximately a million. For the H and K lines, which are too strong to be calibrated, the factor must be much greater.

The weakest lines which can be studied on ordinary stellar spectrograms are of approximately intensity 1 on Rowland's scale. From these to the strongest ordinary solar lines, the number of atoms increases by a factor of about 5000. For such lines as H and K in the later spectral types, or for the Balmer series in the A stars, the factor must be many times greater. Application of the results here derived to the study of spectra will be given in later communications.

CARNEGIE INSTITUTION OF WASHINGTON
MOUNT WILSON OBSERVATORY
February 1928

# PRELIMINARY RESULTS OF A NEW METHOD FOR THE ANALYSIS OF STELLAR SPECTRA[1]

By WALTER S. ADAMS and HENRY NORRIS RUSSELL[2]

ABSTRACT

*A new method of analyzing stellar spectra.*—The calibration of Rowland's intensity scale (*Contribution* No. 358) leads to a knowledge of the relative numbers of atoms in the solar atmosphere which are active in producing different lines in the same spectral region. By assuming thermodynamic equilibrium and that atoms at different levels are equally effective in producing a line, the investigation can be extended to stellar atmospheres in general. The resulting equation (11) connects the relative numbers of atoms producing the same line in different stars with the relative numbers of normal neutral atoms, the excitation potential, the state of ionization, and the electron pressures and the temperatures of the stars.

The application of this relation to high-dispersion spectrograms of typical representatives of the spectral sequence, taken with the coudé spectrograph of the 100-inch reflector, leads to the following provisional results: There appears to be a serious departure from the condition of thermodynamic equilibrium in stellar atmospheres, in that the relative numbers of atoms in excited states, especially those of high energy, are much greater than indicated by theory. The abnormal intensity of the hydrogen lines appears to be a consequence of the phenomenon. The anomaly does not appear to affect ionization, at least not to the extent that it does excitation. Correction for it leads to a new method of determining stellar temperatures, which gives results agreeing closely with the accepted scale. The behavior of enhanced lines of iron, titanium, etc., in red giant stars suggests that these lines are produced largely in extensive chromospheres, supported by radiation pressure. The amount of metallic vapor above a given area of the photosphere varies from something like a hundred times that in the sun (Betelgeuse, Antares) to something less than that in the sun (Sirius), a result in fair agreement with the accepted theory which attributes the photosphere to general opacity in atmospheres of similar constitution.

## 1. INTRODUCTION

The calibration of Rowland's scale of intensity for solar lines[3] makes it possible to deduce from the empirical estimates the relative numbers of atoms in the solar atmosphere which are active in the production of different lines in the same spectral region. Preliminary results of the extension of this method to stellar spectra are discussed in the present paper.

The dispersion and resolving power of the coudé spectrograph of the 100-inch reflector are such that spectrograms of the brighter

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[3] Russell, Adams, and Moore, *Mt. Wilson Contr.*, No. 358; *Astrophysical Journal*, **68**, 1, 1928.

stars can be obtained which show great numbers of lines clear of blends, even in the later spectral types. By comparison with the solar spectrum taken with the same instrument, the intensities of these stellar lines may be estimated upon Rowland's scale with considerable accuracy. For solar and stellar lines in the same part of the spectrum, it appears to be a reasonable assumption that the numbers of atoms in the atmospheres, above equal areas of the sun's and the star's photosphere, which are engaged in the transitions that produce the given lines, are the same. This is equivalent to assuming that an atom making the transition has the same effect toward the production of the line, whether it is high or low in the atmosphere. The recent work of Unsöld[1] indicates that this is approximately true as regards the *widths* of solar lines, even of H and K (though not for the hydrogen lines, which are abnormally broadened). The *depths* of the lines—the blackness in the middle—evidently demand a more complex theory for their interpretation, which is not yet available.

On high-dispersion stellar spectrograms, which are very rarely in danger of overexposure, the width of a line must be much more influential than its depth in usual estimates of the intensity. The assumption stated above appears therefore likely to be safe for most lines, though risky for such exceptional objects as the hydrogen lines. It will be adopted in the discussion which follows.

## 2. THEORETICAL RELATIONS

The number of atoms in any given state of ionization and excitation in a particular point of a star's atmosphere may be predicted from thermodynamic theory if the composition of the gas is known.

According to R. H. Fowler,[2] the numbers of atoms in various energy-states in a gas in thermodynamic equilibrium are defined by the equations

$$\frac{NM_{r+1}}{VM_r} = \frac{(2\pi mkT)^{3/2}}{h^3}\,\frac{u_{r+1}(T)}{u_r(T)}\,e^{-\frac{\chi_r}{kT}-J+L}, \tag{1}$$

$$\frac{(M_r)_s}{M_r} = \frac{1}{u_r(T)}\left\{(\hat{\omega}_r)_s e^{-\{\chi_r-(\chi_r)_s\}/kT-N v_s/V}\right\}. \tag{2}$$

[1] *Zeitschrift für Physik*, **46**, 765, 1927.

[2] *Monthly Notices, R.A.S.*, **85**, 941, 1925; *Philosophical Magazine*, **1**, 845, 1926.

Here $m$, $k$, $T$, $h$ have their usual meanings; $N$ is the number of free electrons in the volume $V$; $M_r$ and $M_{r+1}$ are the numbers of atoms (of a given kind) in this volume which are in the $r$th and $r+1$st states of ionization, i.e., which have lost $r$ or $r+1$ electrons; $\chi_r$ is the energy, in ergs per atom, requisite for the removal of the $r+1$st electron from the normal state of the atom which has lost $r$ electrons; $u_r$ and $u_{r+1}$ are factors taking account of the distribution of the atoms among the different stationary states (we do not require their detailed values here, though Fowler gives them); also, $(M_r)_s$ is the number of atoms in the $r$th state of ionization and the $s$th excited stationary state; $(\hat{\omega}_r)_s$ is the statistical weight of this state, which is equal to the number of component levels into which the corresponding spectroscopic term is resolved in the Zeeman effect; and $(\chi_r)_s$ is the energy necessary for the removal of the $r+1$st electron from an atom in this state. Finally, $J$, $L$, and $v_s$ are quantities which are negligible under the conditions prevailing in stellar atmospheres—under which, also, the quantities $u_r$ are usually nearly equal to the statistical weights $(\hat{\omega}_r)_s$ of the normal energy states (combining the components of a multiplet).

Consider now a particular layer in a star's atmosphere, where the temperature is $T$ and the electron pressure $p'_e$. By the gas law, $p'_e = \frac{N}{V} kT$. Hence, by dropping the insensible terms and introducing the numerical values of $h$, $k$, and $m$, equation (1) becomes

$$p'_e \frac{M_{r+1}}{M_r} = 0.332 T^{5/2} \frac{u_{r+1}}{u_r} e^{-\frac{\chi_r}{kT}}.$$

Here $p'_e$ is measured in dynes /cm², and $\chi$ in ergs per atom. If $p_e$ is the electron pressure in atmospheres and $I_r$ the ionization potential in volts, this becomes

$$\frac{M_{r+1}}{M_r} = 3.28 \times 10^{-7} \frac{T^{5/2}}{p_e} \frac{u_{r+1}}{u_r} e^{-\frac{11{,}600\, I_r}{T}}. \qquad (3)$$

Similarly, if $E_{rs}$ denotes the excitation potential in volts of the $s$th stationary state, measured from the lowest energy state of the $r$th degree of ionization, (3) may be written

$$\frac{(M_r)_s}{M_r} = \frac{(\hat{\omega}_r)_s}{u_r} e^{-\frac{11{,}600\, E_{rs}}{T}}. \qquad (4)$$

Multiplying together the equations (3) for all degrees of ionization up to the $r$th, and also equation (4), and setting for brevity

$$\left.\begin{aligned} &I_0+I_1+\cdots\cdots I_{r-1}+E_{rs}=F_{rs}\,,\\ &3.28\times10^{-7}\frac{T^{5/2}}{p_e}=d\,,\end{aligned}\right\} \tag{5}$$

we find

$$\frac{(M_r)_s}{M_0}=\frac{(\hat{\omega}_r)_s}{u_0}\,e^{-\frac{11{,}600\,F_{rs}}{T}}\,d^r\,. \tag{6}$$

This equation gives the number of atoms in any specified state, in terms of the number of neutral atoms, $M_0$. It is evident that $F_{rs}$ represents the energy required to change an atom from the normal neutral state to that under consideration. This differs for each state, but $d$ is constant at any given point in the atmosphere.

Consider now a single line which is absorbed by atoms in the $r$, $s$ state. Only a certain fraction $x$ of all such atoms will absorb this particular line. This fraction probably depends on quantum relations, though the details are not yet worked out except for the simplest cases. We will assume, then, that for a given line $x$ is a constant, independent of local conditions.

If $n_x$ is the number of atoms in the given volume which are effective in producing the given line, $n_x=x(M_r)_s$. If $n_0$ is the number of neutral atoms in the normal state of lowest energy, and $\hat{\omega}_0$ the weight of this, then $n_0=\frac{\hat{\omega}_0}{u_0}M_r$. Substituting in (6) and setting $\frac{(\hat{\omega}_r)_s}{\hat{\omega}_0}=W_{rs}$, we find

$$n_x=n_0xW_{rs}e^{-\frac{11{,}600\,F_{rs}}{T}}\,d^r\,. \tag{7}$$

Consider now the whole depth of the atmosphere, and integrate along a column of unit cross-section extending from the photosphere to the outer boundary of the atmosphere (the obvious vagueness of these limits reflects the approximate character of our fundamental assumption). The whole number of atoms which are effective in producing the given line will be $N_x=\int n_x dh$, and the number of normal neutral atoms $N_0=\int n_0 dh$.

The quantities $T$ and $d$ vary with the depth $h$. It is probable that $T$ changes relatively little, and certain that $d$ varies greatly. As a first approximation, we may treat $T$ as constant. We may then define $D$ by the equation

$$\frac{N_x}{N_0}=xW_{rs}e^{-\frac{11{,}600\,F_{rs}}{T}}D^r, \tag{8}$$

which makes $D^r$ a sort of weighted mean of the individual values of $d^r$. There will be some level, part way up in the atmosphere, for which $d=D$. If $P_e$ is the electron pressure at this representative level, we shall have

$$P_e=3.28\times10^{-7}T^{5/2}/D\,. \tag{9}$$

Under the isothermal conditions here assumed, $D$ and $P_e$ will be the same for all the lines of a given element in a given state of ionization. For other elements, or for the same element in another state of ionization, they may be very different.

Taking logarithms to the base 10, we find

$$\log N_x=\log N_0+\log xW_{rs}-\frac{5040F_{rs}}{T}+r\log D\,. \tag{10}$$

If a closer approximation should be attempted by taking into account the variation of temperature within the atmosphere, we may define a mean temperature $T_x$ for any arc line (for which the factor involving $d$ does not appear) so that equation (8) is satisfied. For lines of different excitation potential, $T_x$ may be somewhat different; but the adoption of a single general mean value should give a working approximation.

For the singly ionized atom, a comparison of lines of different excitation potential should give, in similar fashion, temperatures $T'_x$, for which again a mean value may be substituted (which need not be the same as the best mean value for the neutral atom).

Substitution in (10) then gives values of $D$, for which again a general mean may be found; and so on for higher degrees of ionization.

Another complication which has to be considered in the actual

problem is the mixture of light from all parts of the star's disk. This demands another integration with respect to the angle of emergence of the rays, which will introduce further complications into the determination of mean values of $T$ and $D$. To discuss these matters in detail would be unprofitable for the present purpose; we must be content with the conclusion that an equation of the form (10) with suitably chosen constants is likely to give an approximation sufficient for use in conjunction with our fundamental assumption concerning the interpretation of the observed lines.

Before proceeding to the application of this equation, something may be said about the type of mean value represented by $P_e$ in the case of an isothermal atmosphere. We have by (5) and (7), $n_0 K = n_x p_e^r$, where $K$ is a constant, while by (8), $N_0 K = N_x P_e^r$. Integrating the first equation over the atmosphere, we find at once

$$N_x P_e^r = \int n_x p_e^r dh .$$

For the first ionization ($r = 1$), $P_e$ is a mean of the electron pressures in different regions, weighted in proportion to the numbers of ionized atoms in these regions; for the second, it is a mean-square value, weighted according to the number of doubly ionized atoms, and so on.

In an atmosphere uniform in composition as well as in temperature, the values of $P_e$ would differ for elements of easy and difficult ionization; but more important differences are likely to arise from the differential effect of radiation pressure in raising some elements into the chromosphere more than others. A chromospheric element will evidently give a low value for $P_e$; but the application of our equation becomes precarious here, since the chromosphere is probably not in thermodynamic equilibrium.[1]

### 3. RESULTING EQUATIONS

In equation (10) the potential $F_{rs}$ is known from analysis of the spectrum, and the statistical ratio $W_{rs}$ from quantum theory. The probability $x$ that the atom will absorb this particular line (among many possibilities) has been calculated theoretically in a few cases, e.g., for the Balmer series. When further theoretical developments

[1] Milne, *Observatory*, **51**, 39, 1928.

permit it to be found in the general case, it will be possible to predict the relative numbers of atoms concerned in the production of all the lines of a given element. By working backward from the values of $N_x$ derived from the observed line-intensities for any one element, $T$, $D$, and $P_e$ could be found, provided that lines belonging to several excitation potentials and to two successive degrees of ionization were available.

This is at present impracticable, and we must have recourse to the comparison of the same lines in different stars. For any two stars we have

$$\log N_x = \log N_0 + \log xW_{rs} - 5040\,\frac{F_{rs}}{T} + r \log D\,,$$

$$\log N'_x = \log N'_0 + \log xW_{rs} - 5040\,\frac{F_{rs}}{T'} + r \log D'\,,$$

whence

$$\log \frac{N'_x}{N_x} = \log \frac{N'_0}{N_0} + 5040\,F_{rs}\left(\frac{1}{T} - \frac{1}{T'}\right) + r \log \frac{D'}{D}\,. \qquad (11)$$

For arc lines, $r=0$, and the last term disappears.

The values of $\log N'_x/N_x$ may be found from the observed spectra. If they are plotted against the excitation potentials $F_{rs}$, the arc lines should give points lying on a straight line of slope proportional to the difference of the inverse temperatures of the stars. If $T$ is known, as for the sun, $T'$ may be found. The intercept of this line on $F_{rs}=0$ gives $\log N'_0/N_0$, which determines the relative abundance of neutral atoms in their normal state in the two atmospheres.

The enhanced lines should give points lying on a line of similar slope, but displaced vertically by $\log D'/D$. From this last quantity the ratio of the effective electron pressures for the given element in the two stars may be found.

Alternately, if the enhanced lines are plotted against their excitation potentials, $E_{rs}$, measured from the normal state of the ionized atom, the intercept for $E_{rs}=0$ will give the relative numbers of ionized atoms in the normal state in the two atmospheres.

It may be emphasized in conclusion that these results depend on two assumptions; that atoms at different levels in the atmosphere

are equally effective in producing a line, and that the relative numbers of atoms in different states are in thermodynamic equilibrium. Departure from these conditions may lead to apparently discordant results.

## 4. OBSERVATIONAL DATA

The present discussion is based on spectrograms obtained at the coudé focus of the 100-inch telescope. The spectrograph is of the auto-collimating type with a focal length of 15 feet, and is placed in the constant-temperature room south of the telescope. The collimator and camera lens is a Ross triplet of 6-inch aperture, and the prism through which the light is returned by a plane mirror is of dense flint with an angle of 52°.5. The field of the camera lens is excellent, and with suitable inclination of the plates and the use of appropriate emulsions it has been found possible to photograph on two strips of plate the entire region from λ 4100 to λ 6800. The linear scale of the negatives at $H\gamma$ is about 2.9 A to the millimeter.

Among the stars for which spectrograms were available, several were selected covering the group from class M to class A—α Orionis, α Scorpii, α Boötis, γ Cygni, α Persei, α Canis Minoris, and α Canis Majoris. All these stars, except Sirius and Procyon, are giants, and some of them are conspicuous supergiants (*c* stars).

The estimates of intensity of the spectral lines were made visually with the aid of standard solar negatives obtained with the same spectrograph. These were placed on a Hartmann spectrocomparator and the solar and stellar spectra compared directly. The range in intensity of the solar lines is sufficient to make comparisons of this kind reasonably satisfactory in the case of stars of the later types, although they cannot, of course, compare in precision with microphotometric methods, with which we hope to continue the investigation. The comparison with the solar spectrum has aided greatly in the elimination of blended lines. In spectra of earlier types, in which the character of the lines differs in some cases notably from that in the solar spectrum, the precision of the estimates is somewhat lower.

Lines known to be seriously blended were excluded from the discussion, but it is probable that many minor blends have been overlooked. The total number of lines observed is, however, so large

## TABLE I

OBSERVED VALUES OF $Y = \log N_x'/N_x$*

| ELEMENT | $E_x$ | α ORIONIS | | α SCORPII | | α BOÖTIS | | γ CYGNI | | α PERSEI | | α CAN. MIN. | | α CAN. MAJ. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | No. | Y | No. | Y | No. | Y | No. | Y | No. | Y | No. | Y | No. | Y |
| *Fe* | 0.06 | 9 | 2.05 | 5 | 2.30 | 10 | 1.17 | 9 | 0.17 | 8 | −0.87 | 9 | −0.90 | 9 | −2.66 |
| | 0.94 | 21 | 1.28 | 19 | 1.23 | 21 | 1.00 | 19 | .27 | 13 | − .50 | 13 | − .77 | 13 | −2.13 |
| | 1.54 | 17 | 0.54 | 11 | 0.97 | 18 | 0.61 | 15 | .27 | 12 | − .40 | 13 | − .60 | 13 | −2.30 |
| | 2.17 | 20 | .30 | 10 | .47 | 21 | .57 | 21 | .25 | 20 | − .70 | 20 | − .50 | 18 | −2.00 |
| | 2.56 | 17 | − .06 | 16 | − .05 | 17 | .36 | 17 | .17 | 17 | − .63 | 17 | − .50 | 17 | −2.00 |
| | 2.89 | 23 | − .19 | 21 | − .03 | 23 | .30 | 22 | .03 | 19 | − .57 | 21 | − .53 | 20 | −1.85 |
| | 3.24 | 13 | − .33 | 12 | − .10 | 12 | − .11 | 11 | .10 | 13 | − .50 | 13 | − .30 | 13 | −1.97 |
| | 3.47 | 19 | − .54 | 16 | − .27 | 19 | .11 | 18 | .03 | 16 | − .70 | 17 | − .33 | 17 | −1.77 |
| | 4.03 | 14 | − .76 | 12 | − .40 | 13 | − .13 | 13 | − .23 | 12 | − .77 | 13 | − .47 | 13 | −1.77 |
| | 4.50 | 13 | −0.87 | 10 | −0.77 | 12 | 0.03 | 12 | 0.03 | 11 | −0.40 | 11 | −0.17 | 12 | −1.60 |
| | ..... | 166 | ±0.39 | 132 | ±0.42 | 166 | ±0.29 | 157 | ±0.36 | 141 | ±0.25 | 147 | ±0.24 | 145 | ±0.40 |
| *Ti* | 0.03 | 23 | 3.15 | 16 | 3.05 | 23 | 2.33 | 16 | 0.10 | 9 | −0.96 | 10 | −1.08 | 9 | −2.00 |
| | 0.83 | 13 | 1.50 | 13 | 1.17 | 25 | 1.27 | 23 | 0.03 | 14 | − .90 | 14 | −0.83 | 15 | −2.11 |
| | 1.02 | 15 | 2.07 | 14 | 1.93 | 12 | 1.80 | 7 | 0.07 | 6 | − .80 | 6 | − .69 | 5 | −1.07 |
| | 1.44 | 14 | 1.80 | 10 | 1.91 | 18 | 1.57 | 10 | − .13 | 5 | −0.64 | 5 | − .58 | 5 | −1.13 |
| | 1.82 | 19 | 1.27 | 18 | 1.48 | 21 | 1.41 | 15 | − .18 | 5 | −1.02 | 5 | − .92 | 5 | −1.82 |
| | 2.09 | 10 | 1.27 | 8 | 1.28 | 10 | 1.15 | 7 | .05 | 2 | −0.27 | 2 | − .27 | 1 | −1.00 |
| | 2.26 | 16 | 1.08 | 14 | 1.12 | 17 | 1.07 | 7 | − .28 | 7 | −0.42 | 7 | −0.30 | 7 | −0.90 |
| | 2.50 | 8 | 0.83 | 7 | 0.92 | 8 | 1.17 | 2 | 0.00 | ..... | ..... | ..... | ..... | ..... | ..... |
| | ..... | 118 | ±0.55 | 100 | ±0.51 | 134 | ±0.47 | 87 | ±0.41 | 48 | ±0.36 | 49 | ±0.37 | 47 | ±0.42 |
| *Ca* | 0.00 | 2 | 2.23 | 2 | 1.80 | 2 | 1.58 | 3 | 0.18 | 1 | −0.23 | 1 | 0.00 | 1 | −3.30 |
| | 1.88 | 8 | 0.13 | 7 | 0.38 | 13 | 0.28 | 12 | .47 | 9 | −0.35 | 13 | −0.42 | 11 | −2.50 |
| | 2.51 | 18 | .00 | 16 | .32 | 16 | .25 | 16 | .17 | 10 | −1.15 | 12 | −1.08 | 11 | −1.93 |
| | 2.81 | 6 | −0.28 | 4 | −0.13 | 6 | 0.10 | 6 | −0.25 | 3 | −0.88 | 5 | −0.41 | 5 | −1.37 |
| | ..... | 34 | ±0.40 | 29 | ±0.49 | 37 | ±0.28 | 37 | ±0.32 | 23 | ±0.33 | 31 | ±0.42 | 28 | ±0.32 |
| *Mg* | 0.00 | 1 | 1.2 | 1 | 1.1 | 1 | 0.9 | 1 | 0.0 | 1 | −1.5 | 1 | −1.5 | 1 | −3.10 |
| | 2.70 | 2 | 0.00 | 3 | 0.10 | 2 | .55 | 3 | − .04 | 2 | −1.10 | 2 | −0.22 | 2 | −1.93 |
| | 4.33 | 3 | −0.80 | 3 | −0.80 | 3 | 0.18 | 2 | +0.20 | 2 | −0.73 | 3 | −0.98 | 3 | −2.07 |
| | ..... | 6 | ..... | 7 | ..... | 6 | ..... | 6 | ..... | 5 | ..... | 6 | ..... | 6 | ..... |
| *Mn* | 0.00 | 2 | 2.7 | 1 | 2.6 | 2 | 2.17 | 2 | −0.23 | ..... | ..... | 1 | −0.3: | 1 | −1.7 |
| | 2.20 | 10 | 1.20 | 9 | 1.38 | 10 | 1.46 | 10 | .55 | 5 | −0.68 | 5 | − .78 | 4 | −2.33 |
| | 2.94 | 18 | −0.17 | 15 | 0.20 | 20 | 0.17 | 19 | −0.07 | 11 | −1.13 | 16 | −0.97 | 17 | −1.75 |
| | ..... | 30 | ±0.55 | 25 | ±0.63 | 32 | ±0.54 | 31 | ±0.40 | 16 | ±0.30 | 22 | ±0.41 | 22 | ±0.35 |
| *Cr* | 0.00 | ..... | ..... | ..... | ..... | 3 | 0.48 | 3 | 0.13 | 2 | 0.40 | 3 | −0.03 | 3 | −2.50 |
| | 0.99 | 24 | 1.33 | 17 | 1.43 | 29 | .97 | 24 | .10 | 16 | −0.80 | 18 | −0.83 | 19 | −2.60 |
| | 2.89 | 2 | 0.27 | 2 | 0.27 | 2 | 0.27 | 1 | −0.9 | ..... | ..... | 1 | −1.5 | 1 | −1.2 |
| | ..... | 26 | ±0.59 | 19 | ±0.57 | 34 | ±0.63 | 28 | ±0.43 | 18 | ±0.81 | 22 | ±0.42 | 23 | ±0.64 |
| *V* | 0.03 | 14 | 2.70 | 6 | 2.63 | 11 | 2.23 | 7 | −0.01 | 2 | −0.50 | 2 | −0.50 | 2 | −2.15 |
| | 0.28 | 26 | 2.20 | 20 | 2.42 | 26 | 2.38 | 13 | − .08 | 3 | −1.27 | 8 | − .64 | 5 | −2.39 |
| | 1.08 | 22 | 2.20 | 23 | 2.27 | 22 | 1.73 | 6 | −0.03 | ..... | ..... | 2 | −0.70 | ..... | ..... |
| | 2.07 | 2 | 1.97 | 1 | 2.4 | 1 | 2.4: | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... |
| | ..... | 64 | ±0.52 | 50 | ±0.51 | 60 | ±0.45 | 26 | ±0.39 | 5 | ..... | 12 | ..... | 7 | ..... |

* All values of $Y$ are positive unless preceded by a minus sign.

TABLE I—*Continued*

| Element | $E_x$ | α Orionis | | α Scorpii | | α Boötis | | γ Cygni | | α Persei | | α Can. Min. | | α Can. Maj. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | No. | $V$ | No. | $V$ | No. | $V$ | No. | $V$ | No. | $V$ | No. | $V$ | No. | $V$ |
| *Sc* | 0.00 | 3 | 3.30 | 2 | 3.23 | 2 | 2.50 | .... | ..... | ..... | ..... | ..... | ..... | ..... | ..... |
| | 1.44 | 5 | 2.33 | 3 | 1.77 | 4 | 1.70 | .... | ..... | ..... | ..... | ..... | ..... | ..... | ..... |
| | 1.93 | 2 | 1.87 | 3 | 1.43 | 2 | 1.43 | .... | ..... | ..... | ..... | ..... | ..... | ..... | ..... |
| | ..... | 10 | ..... | 8 | ..... | 8 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... |
| *Na* | 0.00 | 2 | * | 2 | * | 2 | * | 2 | * | 2 | −0.38 | 2 | −0.25 | 2 | −1.10 |
| | 2.10 | 4 | 0.27 | 2 | 0.50 | 7 | 0.28 | 5 | 0.23 | 1 | −2.0 | 3 | −0.67 | 4 | −1.47 |
| | ..... | 6 | ..... | 4 | ..... | 9 | ..... | 7 | ..... | 3 | ..... | 5 | ..... | 6 | ..... |
| *Fe*+ | 2.83 | 20 | −0.90 | 16 | −0.44 | 24 | −0.40 | 21 | 2.16 | 20 | 1.33 | 20 | 0.24 | 20 | −0.18 |
| | 3.56 | 12 | −1.14 | 9 | −0.79 | 12 | −0.49 | 11 | 1.90 | 8 | 0.66 | 10 | 0.30 | 11 | 0.54 |
| | ..... | 32 | ±0.39 | 25 | ±0.44 | 36 | ±0.23 | 32 | ±0.38 | 28 | ±0.45 | 30 | ±0.48 | 31 | ±0.26 |
| *Ti*+ | 1.16 | 15 | −0.03 | 10 | 0.35 | 19 | 0.41 | 13 | 1.60 | 14 | 1.07 | 17 | 0.48 | 17 | −1.05 |
| | 1.57 | 6 | .17 | 4 | .58 | 6 | .37 | 4 | 1.77 | 5 | 0.13 | 6 | .10 | 6 | −0.78 |
| | 2.00 | 4 | − .17 | 3 | .03 | 4 | .40 | 4 | 2.33 | 3 | 1.43 | 3 | .83 | 3 | .08 |
| | 2.93 | 3 | −0.23 | 3 | −0.42 | 3 | 0.18 | 2 | 2.20 | 2 | 1.23 | 2 | 0.00 | 2 | −0.75 |
| | ..... | 28 | ..... | 20 | ..... | 32 | ..... | 23 | ..... | 24 | ..... | 28 | ..... | 28 | ..... |
| *Ba*+ | 0.00 | 2 | 1.43 | 2 | 1.27 | 2 | 0.87 | 2 | 1.20 | 2 | 0.77 | 2 | 0.30 | 2 | −2.23 |
| | 0.65 | 2 | 0.90 | 2 | 0.90 | 2 | .12 | 2 | 1.13 | 2 | −0.25 | 2 | −1.00 | 2 | −1.67 |
| | 2.50 | 1 | 0.0 | 1 | 1.1 | 1 | 0.3 | .... | ..... | 1 | 0.5 | 1 | −0.5 | 1 | −0.5 |
| | ..... | 5 | ..... | 5 | ..... | 5 | ..... | 4 | ..... | 5 | ..... | 5 | ..... | 5 | ..... |
| *Sc*+ | 0.31 | .... | ..... | .... | ..... | 1 | 0.7 | 1 | 1.0 | .... | ..... | .... | ..... | .... | ..... |
| | 0.60 | 3 | 0.98 | 3 | 0.52 | 7 | .93 | 4 | 1.63 | 5 | 1.30 | 7 | 0.13 | 6 | −1.48 |
| | 1.50 | 10 | 0.20 | 7 | 0.11 | 11 | 0.58 | 8 | 2.15 | 5 | 0.92 | 6 | −0.07 | 6 | −0.73 |
| | ..... | 13 | ..... | 10 | ..... | 19 | ..... | 13 | ..... | 10 | ..... | 13 | ..... | 12 | ..... |
| *Y*+ | 0.13 | 1 | 2.6 | 1 | 2.6 | 2 | 1.13 | 2 | 2.80 | .... | ..... | .... | ..... | .... | ..... |
| | 1.03 | 3 | 1.77 | 4 | 1.10 | 5 | 0.30 | 4 | 1.58 | 3 | 0.98 | 3 | 0.27 | .... | ..... |
| | 1.73 | 4 | 1.40 | .... | ..... | 3 | 1.00 | 3 | 2.28 | .... | ..... | .... | ..... | .... | ..... |
| | ..... | 8 | ..... | 5 | ..... | 10 | ..... | 9 | ..... | 3 | ..... | 3 | ..... | .... | ..... |

* Too strong to calibrate.

that the inclusion of such blends should have little influence on the further discussion, except to exaggerate the discordance due to the accidental errors of the observations.

From the estimated intensities, the relative numbers $N$ of atoms involved in the production of these lines were then derived by the aid of the calibration tables discussed in *Contribution* No. 358,[1] and the values of log $N'_x/N_x$ tabulated for all the lines (with the sun as standard).

The results, which are much too voluminous to give *in extenso*, are summarized in Table I, in which the lines of each element are grouped into means according to the value of the excitation po-

[1] *Astrophysical Journal*, **68**, 1, 1928.

tential $E_x$. In some cases these groups consist entirely of lines originating in the same spectroscopic term, but for most of the higher lines two or more closely adjacent terms are combined. The groups, however, are sufficiently similar to permit the use of the same mean value of $E_x$ for all the stars in the table.

For each star the column headed "No." gives the number of lines combined to form the mean, and that headed $Y$, the mean value of log $N'_x/N_z$, which will hereafter be called $Y$ for brevity. The lowest line for each element gives the whole number of lines which are here studied, not counting those rejected because of blending, etc.; and, also, in the $Y$ column, the average deviation of the values of $Y$ given by a single line from the group-means in the column. Groups containing but few lines are omitted in finding these averages.

The average values of a residual for different elements are *Fe*, ±0.33; *Ti*, ±0.34; *Ca*, ±0.37; *Mn*, ±0.45; *Cr*, ±0.58; *V*, ±0.47; $Fe^+$, ±0.37. For the different stars they are α Orionis, ±0.48; α Scorpii, ±0.51; α Boötis, ±0.41; γ Cygni, ±0.38; α Persei, ±0.42; α Can. Min., ±0.39; α Can. Maj., ±0.40; so that the observations for the red stars appear to be somewhat less accurate. The general mean value is ±0.43, corresponding to a probable error of ±0.37 in log $N'_x/N_x$. It was found during the calibration of Rowland's tables[1] that the probable error of the value of log $N$ derived from a single solution was ±0.32. Since the present results involve the comparison of estimates made on two spectra, their accuracy is rather better than was to be expected. Even the best of the values of $Y$ in the table have a probable error of ±0.07; the second decimal place is therefore of no significance—a mere guard figure. The whole number of lines listed for each star ranges from 588 for α Boötis to 329 for α Persei. It is worthy of note that 248 arc lines of various metals were observed in the spectrum of Sirius, as against 76 enhanced lines.

### 5. TEST FOR THERMODYNAMIC EQUILIBRIUM; EMPIRICAL CORRECTIONS

It is obvious on inspection of Table I that the values of $Y$ decrease with increasing $E_x$ for the stars cooler than the sun, and in-

[1] *Mt. Wilson Contr.*, No. 358; *Astrophysical Journal* **68**, 1, 1928.

crease for the hotter stars, as theoretically they should. If the conditions of thermodynamic equilibrium and our other assumptions were exactly true, then, for any one element and one star, $Y$ should be a linear function of $E_x$. Much the best data for testing these

TABLE II

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $E$..... | 1.06 | 0.94 | 1.54 | 2.17 | 2.56 | 2.89 | 3.24 | 3.47 | 4.03 | 4.50 |
| $Y$..... | 2.18 | 1.26 | 0.75 | 0.38 | −0.06 | −0.11 | −0.22 | −0.40 | −0.58 | −0.82 |
| Comp..... | 2.12 | 1.28 | .78 | .32 | .07 | − .12 | − .30 | − .41 | − .62 | − .76 |
| O−C..... | +0.06 | −0.02 | −0.03 | +0.06 | −0.13 | +0.01 | +0.08 | +0.01 | +0.04 | −0.06 |

assumptions are afforded by the long series of iron lines, which extend over a greater range of $E_x$ than any others; and the best stars for a test are evidently $\alpha$ Orionis and $\alpha$ Scorpii. The run of the $Y$'s for these stars is so similar that one may take the mean for the two, to diminish accidental error. The results are given in Table II.

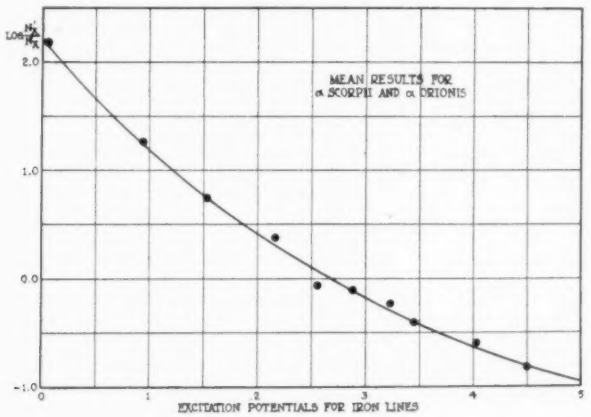


FIG. 1

When plotted (Fig. 1), these give points deviating from a straight line by far more than the amount permitted by observational error, which should average about ±0.10. It is evident that some complicating cause is at work. The deviation is in such a sense that $\Delta Y/\Delta E_x$ diminishes in absolute value as $E_x$ increases. A similar effect, though of smaller magnitude, is indicated on plotting the data for Arcturus. This may be interpreted as indicating that the

number of atoms in highly excited states is greater than it would be in strict thermodynamic equilibrium; and there is independent evidence in favor of this hypothesis. It has long been known[1] that the lines of the Balmer series of hydrogen are far stronger in the cooler stars than appears to be possible if the fraction of atoms in the two-quantum state, which alone are capable of absorbing them, is that given by the equation (4). For these lines, $E_x$ is 10.15 volts, and the fraction of all the hydrogen atoms in this state should be of the order of $10^{-9}$ in the sun and $10^{-17}$ in $\alpha$ Orionis. The actual proportion of excited atoms must be thousands of times as great in the first case, and millions in the second, at a low estimate. Further evidence in support of this view is found in the appearance in stars of class B of lines of *He* I, *O* III, *N* III, etc., which have very high

TABLE III

| Points | $\Delta Y/\Delta E_x$ | Mean $E_x$ | Comp. | O−C |
|---|---|---|---|---|
| 1– 3 | −0.92 | 0.8 | −0.91 | +0.01 |
| 2– 5 | .74 | 1.8 | .73 | − .01 |
| 3– 8 | .58 | 2.5 | .61 | + .03 |
| 5–10 | −0.43 | 3.6 | −0.43 | 0.00 |

excitation potentials, and would, on the elementary theory, demand an enormous abundance of the corresponding atoms in their normal state.

It appears legitimate, therefore, to regard the curvature exhibited in Figure 1 as another illustration of a familiar, though unexplained, phenomenon, and as evidence that the same influence operates, in a milder degree, for low excitation potentials. On this hypothesis, the closest approach to thermodynamic equilibrium should be exhibited by the initial and steepest portion of the curve for low excitation potentials.

The law of operation of the disturbing influence cannot of course be predicted, but a good idea of its behavior over the observed range can be obtained by drawing straight lines to represent successive groups of the plotted points, thus finding values of $\Delta Y/\Delta E_x$ for various mean values of $E_x$. The results are in Table III.

[1] Payne, *Stellar Atmospheres*, pp. 56, 188, 1925.

The linear formula

$$\frac{\Delta Y}{\Delta E_x} = -1.05 + 0.175 E_x$$

represents these data almost perfectly, as shown above. This indicates that $Y$ and $E$ are connected by the quadratic formula

$$Y = A - 1.05 E_x + 0.087 E_x^2$$

With $A = 2.19$ this gives the computed values and residuals of Table II. From these residuals the probable error of a tabular value of $Y$ is $\pm 0.049$. On the average, one of these points depends upon thirty observed lines, so that one might expect a probable error of $\pm 0.068$. The agreement with our formula is therefore too good, but a linear formula would give wholly inadmissible residuals.

This empirical formula can hardly hold good far beyond the observed range, for it would make $Y$ increase with $E_x$ beyond $E_x = 6$; but, over the range of values covered by the present data, it appears to give a good representation of the observed effect.

Whether the effect for stars of other temperatures would follow the same law cannot be determined decisively from the present data, but plots of the values of $Y$ for the iron lines of the other five stars (Fig. 2), with parabolic curves obtained by diminishing the vertical amplitude of the original curve in fixed ratios, show, at least, that the hypothesis that the effect does follow this law is consistent with all the data.

In the ensuing discussion, therefore, it has been assumed that the observed quantity $Y$ is a linear function, not of $E_x$, but of a quantity $X$ defined by

$$X = 1.05 E_x - 0.087 E_x^2 \tag{12}$$

so that

$$Y = Y_0 + SX , \tag{13}$$

and also that, if the influence which disturbs thermodynamic equilibrium were absent, we should find

$$\frac{\Delta Y}{\Delta E_x} = 1.05 S , \tag{14}$$

which is the initial slope of the actual curves for $E_x = 0$.

Any such empirical correction must, in fine, be justified, or otherwise, by its results, and it will soon appear that the results of the present process are satisfactory; but it should be clear from the foregoing that it rests upon definite and reasonable assumptions, and that the numerical coefficients of the interpolation formula which is used to make the actual correction have been derived by an objective process, quite independent of the results which follow from it.

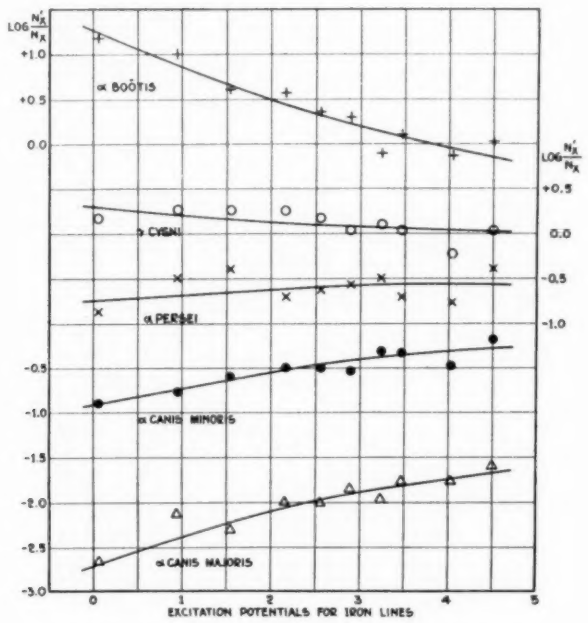


FIG. 2

## 6. DETERMINATION OF STELLAR TEMPERATURES

If, then, we plot the observed values of $Y$ for any given star and a given element against the corresponding values of $X$ from (12), we should obtain straight lines, from which the slope $S$ can be determined. Theoretically, the values of $S$ should be the same for all elements in a given star; practically, the errors of observation will cause discordances, and we must take a weighted mean.

If the $n$ values of $Y$ were distributed uniformly along the extreme observed range $r$ of $X$, the weight of a determination would be pro-

portional to $nr^2$. This should give a fair approximation to the actual weights, except when some important point at one end of the range is weakly determined. Applying this principle to α Orionis, as a typical case, we find the weights given in Table IV.

It is evident that the long series of iron and titanium lines carry nearly all the weight. In some cases (*Cr*, *V*) the unequal distribution of the points greatly reduces the weight. For the enhanced lines, the available range of excitation potential is small, and nothing

TABLE IV

| Element | $n$ | $r$ | $nr^2$ | Adopted Weight |
|---|---|---|---|---|
| *Fe* | 166 | 2.95 | 1460 | 10 |
| *Ti* | 118 | 2.08 | 510 | 5 |
| *Ca* | 34 | 2.27 | 180 | 1 |
| *Mg* | 6 | 2.90 | 50 | 1/2 |
| *Mn* | 30 | 2.34 | 160 | 1/2 |
| *Cr* | 26 | 1.35 | 50 | 0 |
| *V* | 54 | 2.07 | 230 | 0 |
| *Ba*+ | 5 | 2.08 | 20 | 0 |
| *Y*+ | 7 | 1.48 | 15 | 0 |
| *Ti*+ | 28 | 1.24 | 40 | 0 |

would be gained by introducing them into the mean, although this would not be the case for $Ti^+$ if the ultra-violet lines were observable. The weights finally assigned are given in the last column of Table IV. The testimony of independent elements has been assigned somewhat greater importance than a formal theory indicates. For α Persei and the hotter stars, *Ca* and *Mn* are dropped, as the data are poor, and *Ti* receives lower weight.

The resulting values of the slope $S$ derived from the various elements, with the adopted mean values, are given in Table V. The agreement of the results derived from different elements is good. The final mean values differ little from those which would have been derived from the iron lines alone.

From these results the stellar temperatures may immediately be derived. From equations (14) and (11) we have

$$5040\left(\frac{1}{T}-\frac{1}{T'}\right)=1.05\,S\,.$$

Here $T'$ is the temperature of the star's atmosphere, and $T$ that of the sun's. Following Eddington,[1] we may take this as $0.88\,T_e$, where $T_e$ is the effective temperature of the photosphere. Taking $T_e$ at the round value 6000° for the sun, we find the values given in Table VI. The agreement of the temperatures thus determined with those familiar from other considerations is very satisfactory.

TABLE V

SLOPE $S=\Delta Y/\Delta X$

| Element | α Orionis | α Scorpii | α Boötis | γ Cygni | α Persei | α Can. Min. | α Can. Maj. |
|---|---|---|---|---|---|---|---|
| *Fe* | −1.02 | −1.01 | −0.49 | −0.08 | +0.07 | +0.21 | +0.32 |
| *Ti* | 0.98 | 0.83 | .53 | − .02 | .23 | .20 | .50 |
| *Ca* | 1.06 | .82 | .62 | .00 | ..... | ..... | ..... |
| *Mg* | 0.70 | .65 | .22 | + .02 | +0.30 | +0.30 | +0.30 |
| *Mn* | −1.10 | −0.90 | −0.70 | +0.10 | ..... | ..... | ..... |
| Mean | −1.00 | −0.93 | −0.51 | −0.07 | +0.10 | +0.21 | +0.35 |

TABLE VI

| Star | $S$ | $5040/T'$ | Atmosphere $T$ | Photosphere $T_e$ | Spectrum |
|---|---|---|---|---|---|
| α Orionis | −1.00 | 2.00 | 2520° | 2900° | cM2 |
| α Scorpii | − .93 | 1.93 | 2620 | 3000 | cM1 |
| α Boötis | − .51 | 1.49 | 3380 | 3850 | gK0 |
| γ Cygni | − .07 | 1.02 | 4950 | 5600 | cF8 |
| (Sun) | ( .00) | (0.95) | (5300) | (6000) | dG0 |
| α Persei | + .10 | .85 | 5940 | 6700 | cF5 |
| α Can. Min | + .21 | .73 | 6900 | 7850 | dF4 |
| α Can. Maj. | +0.35 | 0.58 | 8700 | 9900 | A2s |

The spectra in the last column, which are given for comparison, are from Mount Wilson determinations. The well-known difference of temperature between dwarfs and giants appears to extend at least as high as class F5.

## 7. ABUNDANCE OF THE DIFFERENT ELEMENTS

It follows from equations (13) and (11) that for arc lines $Y_0 = \log N_0'/N_0$; that is, $Y_0$ gives the relative numbers of atoms of the

[1] *The Internal Constitution of the Stars*, p. 358, 1926.

given element in their normal neutral state above equal areas of the photosphere in the two stars. With sufficient approximation for our purpose, this may be taken as the value of the total number of neutral atoms in the two cases. For enhanced lines, $Y_0$ gives the relative numbers of ionized atoms. Every entry in Table I gives a value of $Y_0 = Y - SX$. Taking means of the values for different groups of lines of the same element, weighted according to the number of lines, we obtain the mean values given in Table VII.

TABLE VII

$\text{Log}\ \frac{N_0'}{N_e}$*

RELATIVE ABUNDANCE OF DIFFERENT ELEMENTS IN STELLAR ATMOSPHERES

| Element | α Orionis | α Scorpii | α Boötis | γ Cygni | α Persei | α Can. Min. | α Can. Maj. |
|---|---|---|---|---|---|---|---|
| *Fe* | 2.10 | 2.12 | 1.39 | 0.27 | −0.81 | −0.92 | −2.74 |
| *Ti* | 2.96 | 2.83 | 2.14 | 0.03 | −0.89 | −1.07 | −2.06 |
| *Ca* | 2.02 | 2.05 | 1.25 | 0.33 | −0.91 | −1.05 | −2.76 |
| *Mg* | 1.96 | 1.88 | 1.65 | 0.12 | −1.29 | −1.28 | −2.96 |
| *Mn* | 2.51 | 2.66 | 1.77 | 0.26 | −1.21 | −1.34 | −2.60 |
| *Cr* | 2.30 | 2.32 | 1.35 | 0.13 | −0.75 | −0.94 | −2.83 |
| *V* | 2.84 | 2.97 | 2.39 | −0.02 | −0.94 | −0.70 | −2.37 |
| *Sc* | 2.96 | 3.06 | 2.38 | ..... | ..... | ..... | ..... |
| *Na* | 2.06 | 2.18 | 1.21 | 0.35 | −0.99 | −0.60 | −1.86 |
| *Fe*+ | 1.42 | 1.67 | 0.81 | 2.24 | 0.90 | −0.24 | −0.78 |
| *Ti*+ | 1.36 | 1.61 | 1.07 | 1.91 | 0.80 | 0.12 | −1.31 |
| *Ba*+ | 1.60 | 1.73 | 0.80 | 1.17 | 0.29 | −0.53 | −1.94 |
| *Sc*+ | 1.58 | 1.30 | 1.24 | 1.98 | 1.01 | −0.17 | −1.45 |
| *Y*+ | 2.79 | 2.06 | 1.12 | 2.21 | 0.89 | 0.08 | ..... |

* All values of $\text{Log}\frac{N_0'}{N_e}$ are positive unless preceded by a minus sign.

## 8. ATMOSPHERIC PRESSURES; EVIDENCE FOR EXTENSIVE CHROMOSPHERES

From equations (11) and (9) we find, comparing neutral and ionized atoms of the same element,

$$\log \frac{P_e'}{P_e} = \log \frac{N_0'}{N_0} - \log \frac{N_1'}{N_1} + \frac{5}{2} \log \frac{T'}{T} + 5040I\left(\frac{1}{T} - \frac{1}{T'}\right).$$

Here $\log N_0'/N_0$ and $\log N_1'/N_1$ are the relative abundances of neutral and ionized atoms as given in Table VII. The resulting values of the relative electron pressures in the various stars as derived from the three elements for which both arc and enhanced lines are available are given in Table VIII.

The electron pressures thus computed range from $10^{-8}$ of the solar value for α Orionis to two hundred times the solar value for Sirius. Something is evidently wrong. The relative intensities of the arc and spark lines change much less with increasing stellar temperature than might be expected. We are again apparently faced with a serious departure from thermodynamic equilibrium.

It is tempting to assume that the effective ionization potentials are diminished in the same fashion as the effective excitation potentials, but there is good evidence that this is not the case.

TABLE VIII

COMPUTED PRESSURES

| Element | α Orionis | α Scorpii | α Boötis | γ Cygni | α Persei | α Can. Min. | α Can. Maj. |
|---|---|---|---|---|---|---|---|
| *Fe:* | | | | | | | |
| *a*) Terms not including I | −0.12 | −0.32 | +0.09 | −2.05 | −1.59 | −0.41 | −1.40 |
| *b*) Terms including I... | −8.22 | −7.67 | −4.22 | −0.55 | +0.78 | +1.72 | +2.90 |
| log $P_e'/P_e$..... | −8.34 | −7.99 | −4.13 | −2.60 | −0.81 | +1.31 | +1.50 |
| *Ti:* | | | | | | | |
| *a*)..... | +0.50 | +0.45 | +0.58 | −1.96 | −1.67 | −0.82 | −0.41 |
| *b*)..... | −7.14 | −6.66 | −3.68 | −0.48 | +0.68 | +1.50 | +2.52 |
| log $P_e'/P_e$..... | −6.64 | −6.21 | −3.10 | −2.44 | −0.99 | +0.68 | +2.11 |
| *Sc:* | | | | | | | |
| *a*)..... | +0.58 | +0.99 | +0.65 | ..... | ..... | ..... | ..... |
| *b*)..... | −6.90 | −6.43 | −3.46 | ..... | ..... | ..... | ..... |
| log $P_e'/P_e$..... | −6.32 | −5.44 | −2.81 | ..... | ..... | ..... | ..... |

In the solar spectrum the enhanced lines of *Ca*, *Sr*, *Sc*, *Ti*, and other elements of easy ionization are much stronger than the arc lines. It is clear that most of the atoms of these elements in the reversing layer are ionized. This is confirmed by the fact that in sun-spots the arc lines are greatly strengthened and the enhanced lines little affected. Indeed, Unsöld's data[1] derived from the widths of the ultimate lines indicate that the number of ionized atoms is seven hundred times that of the neutral atoms for *Ca*, and two hundred times that for *Sr*. For *Fe*, the arc lines are much stronger in the sun than the enhanced lines. A rough comparison may be made by taking lines of the same excitation potential in the two cases, and comparing the intensities of the strongest lines arising from the corresponding levels. We thus find the results in Table IX.

[1] *Zeitschrift für Physik*, **46**, 772, 1928.

The three groups of lines are remarkably consistent in indicating that the number of neutral atoms involved in their production is about twelve times that of the ionized atom. Neglect of the quantum weights probably affects the conclusion but little; but the factor $x$, which represents the relative probability of the absorption of different multiplets, is a different matter. For the neutral atom the multiplets here considered are among the strongest which arise from the given levels; for the ionized atom there is no doubt that the strongest multiplets are far in the ultra-violet, where the spectrum is incompletely analyzed. The actual number of ionized atoms must therefore be much more than one-twelfth that of the neutral atoms, and may even exceed the latter.

TABLE IX

| *Fe* | | | | *Fe+* | | | | LOG *Fe+/Fe* |
|---|---|---|---|---|---|---|---|---|
| No. | *E* | Mean λ | log $N'_x$ | No. | *E* | Mean λ | log $N'_x$ | |
| 5 | 2.88 | 5028 | 3.17 | 5 | 2.86 | 4753 | 2.03 | −1.14 |
| 4 | 3.27 | 5370 | 2.70 | 4 | 3.20 | 5278 | 1.60 | −1.10 |
| 4 | 3.90 | 5957 | 2.23 | 4 | 3.88 | 6248 | 1.17 | −1.06 |

TABLE X

RELATIVE IONIZATION IN THE SUN

| Element | *Na* | *Sr* | *Ca* | *Sc* | *Ti* | *V* | *Cr* | *Mn* | *Mg* | *Fe* | *Ba+* | *Ca+* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *I* | 5.13 | 5.67 | 6.09 | 6.57 | 6.80 | 6.76 | 6.74 | 7.40 | 7.61 | 7.83 | 9.98 | 11.82 |
| log $N_1/N_0$ | 2.67 | 2.05 | 1.65 | 1.10 | 0.98 | 1.02 | 1.04 | 0.41 | 0.19 | (0.00) | −2.62 | −3.88 |

Iron-arc lines of low excitation potential are strengthened in sunspots, while those of high excitation potential are slightly weakened. The enhanced lines are all greatly weakened in spots.

Equation (10), the quantum weights being neglected, gives

$$\log N_1 = \log N_0 + \log D - 5040\, I/T\ .$$

The relative values of log $N_1/N_0$ for different elements depend only on the ionization potential and the temperature. For a temperature of 5300° these values (*Fe* as a standard) are given in Table X.

With Unsöld's value for *Ca*, it would follow that about 95 per cent of the atoms of *Fe* were ionized. This figure is entirely too high. With the temperature adopted by Unsöld for the reversing layer, 4830°, *Fe* comes out 90 per cent ionized. To account for the facts, if the thermodynamic factor is given its full value, it is necessary to assume that the effective pressure in the solar atmosphere is of the order of ten times greater for *Fe* than for *Ca*, which is quite reasonable, as *Ca* rises far higher in the chromosphere.

The departure from thermodynamic equilibrium which is indicated by Table VIII may, however, reasonably be found in another direction. At the surfaces of all the stars here considered, radiation pressure is much stronger, in comparison to gravity, than it is in the sun (see Table XII). The chromosphere, which is held up by radiation pressure, is not even approximately in thermodynamic equilibrium. The presence of extensive chromospheres upon these stars, especially the redder giants, may explain the discrepancies. It is interesting in this consideration to note that the computed pressure in the stars is lower (that is, the observed strength of the enhanced lines is greater in proportion) for *Fe* than for *Ti* and *Sc*. Those of $Ti^+$ and $Sc^+$ rise much higher in the solar chromosphere than those of $Fe^+$. A passage to conditions in which all three were present in a very extensive chromosphere would produce a greater change for $Fe^+$ than for the others, and this agrees with observation.

The assumption of an extensive chromosphere, however, does not explain the unexpected strength of the arc lines of the metals in Sirius, relative to the enhanced lines, and further study will thus be necessary.

### 9. EXTENT OF THE ATMOSPHERES

In estimating the extent of the atmosphere, it would be more satisfactory to use the whole number of atoms of a given element than the number of ionized or neutral atoms, and this can be done if the relative numbers of atoms of the two sorts in the sun are estimated. For our present purpose, we may start with the assumption, probably near the truth, that *Fe* and $Fe^+$ are equally abundant in the sun. The relative proportions of neutral and ionized atoms of other elements in the sun may then be found from Table X, and those in the stars referred to the whole amount in the sun as stand-

ard, by Table VII. The results are given in Table XI. For each element, the neutral atoms are given in the upper line and the ionized atoms in the lower. The results are given in the nearest round numbers, which is all that the data warrant. It should be borne in mind that the unit of abundance is different for each element, but the same for any one element in all the stars.

TABLE XI

COMPOSITION OF STELLAR ATMOSPHERES

| | *Na* | *Ca* | *Sc* | *Ti* | *V* | *Cr* | *Mn* | *Mg* | *Fe* | *Ba* | *Y* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Sun | 0.002 | 0.02 | 0.07 | 0.09 | 0.09 | 0.08 | 0.28 | 0.39 | 0.50 | 0.00 | 0.08 |
| | .998 | 0.98 | 0.93 | 0.91 | 0.91 | 0.92 | 0.72 | 0.61 | 0.50 | 1.00 | 1.92 |
| α Orionis | .2 | 2 | 70 | 90 | 60 | 20 | 90 | 40 | 60 | ..... | ..... |
| | ..... | ..... | 30 | 20 | ..... | ..... | ..... | ..... | 15 | 40 | 600 |
| α Scorpii | .3 | 2 | 90 | 60 | 80 | 20 | 130 | 30 | 70 | ..... | ..... |
| | ..... | ..... | 20 | 40 | ..... | ..... | ..... | ..... | 20 | 50 | 110 |
| α Boötis | .03 | 0.4 | 15 | 10 | 20 | 2 | 15 | 20 | 10 | ..... | ..... |
| | ..... | ..... | 15 | 10 | ..... | ..... | ..... | ..... | 3 | 6 | 10 |
| γ Cygni | 0.005 | 0.05 | ..... | 0.1 | 0.1 | 0.1 | 0.5 | 0.5 | 1 | ..... | ..... |
| | ..... | ..... | 90 | 70 | ..... | ..... | ..... | ..... | 90 | 15 | 150 |
| α Persei | $2\times10^{-4}$ | $2\times10^{-3}$ | ..... | 0.01 | 0.01 | 0.02 | 0.02 | 0.02 | 0.08 | ..... | ..... |
| | ..... | ..... | 10 | 6 | ..... | ..... | ..... | ..... | 4 | 2 | 8 |
| α Can. Min. | $5\times10^{-4}$ | $2\times10^{-3}$ | ..... | $8\times10^{-3}$ | 0.02 | 0.01 | 0.01 | 0.02 | 0.06 | ..... | ..... |
| | ..... | ..... | 0.6 | 1 | ..... | ..... | ..... | ..... | 0.3 | 0.3 | 1 |
| α Can. Maj. | $3\times10^{-5}$ | $4\times10^{-5}$ | ..... | $8\times10^{-4}$ | $4\times10^{-4}$ | $1\times10^{-4}$ | $7\times10^{-4}$ | $5\times10^{-4}$ | $9\times10^{-4}$ | ..... | ..... |
| | ..... | ..... | 0.03 | 0.04 | ..... | ..... | ..... | ..... | 0.08 | 0.01 | ..... |

If more enhanced lines had been studied, as they will be when observations can be made in the ultra-violet, the table could have been made considerably more complete. As it stands, however, it is very instructive. The percentage of ionization appears considerable, even in the coolest stars. The relative abundance of the elements from *Sc* to *Ba* is much the same in all the stars, except that the figures for *Cr* run rather low. The low values for neutral *Ca* must be due largely to high ionization; but, unless it behaves very differently from *Sc* and *Ti*, which are only a little harder to ionize, it must be present in most of the stars in smaller proportions than in the sun. The H and K lines are unfortunately too strong to be calibrated by the present method. Measures by more precise methods should settle this point.

Sodium must be very greatly ionized, even in the coolest stars (except a few dwarfs). By allowing for this, it appears to be abundant everywhere, and surprisingly so in the hotter stars.

The total amount of metallic vapor above unit area of the photosphere, with the sun as standard, may be estimated from these data as 100 for $\alpha$ Orionis and $\alpha$ Scorpii; 80 for $\gamma$ Cygni; 20 for $\alpha$ Boötis; 6 for $\alpha$ Persei; 0.6 for Procyon; and 0.05 for Sirius. No account has been taken of doubly ionized atoms.

### 10. THEORETICAL EXTENT OF ATMOSPHERE

It is of interest to compare these results with the predictions of theory. Eddington[1] has found a solution which "should give a good approximation for the outer layers" of a stellar atmosphere down to an optical depth $\tau$ of 0.25, which he takes as the top of the photosphere. Within this upper region the temperature ranges only between 0.84 and 0.91 times the effective temperature $T_e$ of the photosphere, and thus may be taken as isothermal. If $g$ is the surface gravity and $p_G$ the gas pressure, he finds that

$$p_G^2 = \text{const. } \tau g T_e^{9/2} .$$

The constant would be the same for all stars, provided that the constant $C$ were the same as in the equation[2] for the general opacity,

$$K = C\rho/\mu T^{7/2} .$$

In the derivation of this equation it has been assumed that all the atoms present are ionized. This is a safe assumption inside a star, but not in the atmosphere of the cooler ones. The neutral atoms produce very little opacity, but make their full contribution to the density. We may therefore regard $C$ as variable and equal to the ratio of the total number of atoms to the number of ionized atoms. Eddington's analysis then gives

$$p_G^2 = \text{const. } C\tau g T_e^{9/2} .$$

Now let $Y_G$ be the mass above unit area, at the depth corresponding to $\tau$, which can be supported by the gas pressure. Then $p_G = gY_G$ and

$$Y_G = \text{const. } C^{1/2}\tau^{1/2} g^{-1/2} T_e^{9/4} .$$

If $Y_r$ is the mass per unit area which could be supported by the pressure of the star's whole radiation, we have

$$Y_R = \text{const. } T_e^4/g , \qquad (15)$$

[1] *The Internal Constitution of the Stars*, pp. 358–362, 1926.

[2] *Op. cit.*, p. 229.

whence

$$Y_G = \text{const. } \tau^{1/2} C^{1/2} Y_R^{1/2} T_e^{1/2} . \qquad (16)$$

This equation shows the amount of matter in the reversing layer above the optical depth $\tau$. It takes no account of the chromosphere, which is supported by selective radiation pressure arising from the spectral lines. If $Z$ is the mass per unit area which is actually so supported, we may write

$$Z = z Y_R . \qquad (17)$$

If the dark lines were distributed uniformly over the whole spectrum, $z$ would be a constant. In practice it will depend on the ratio of the energy in the region where the lines are to the total energy, and will be a factor of the nature of luminous efficiency and different for different elements. It may be taken as roughly proportional to the visual luminous efficiency (the photographic value might be better).

The differences in extent of atmosphere for various stars (down to the same optical depth) should therefore depend mainly upon $Y_R$. Increase in the supporting power of radiation should be accompanied by a more extensive reversing layer, and a still more extensive chromosphere. For the cooler stars, the decrease in the luminous efficiency $z$ should diminish the chromosphere, but the increase in the factor $C$ with diminished ionization should extend the reversing layer. The value of $Y_R$ depends mainly on the absolute magnitude of the star. If $m$ is the mass and $M$ the absolute bolometric magnitude, we find easily

$$\log Y_R = 1.93 - 0.4M - \log m ,$$

the constant being adjusted to make $Y_R = 1$ for the sun. Over the range for $M = -2$ to $M = 10$, Eddington's mass-luminosity relation is represented by

$$\log m = 0.61 - 0.12M,$$

with residuals nowhere exceeding 0.08. Hence, accurately enough for our purpose,

$$\log Y_R = 1.32 - 0.28\, M ,$$

and $Y_R$ is nearly doubled for an increase of absolute brightness by one magnitude.

For the stars here considered, we need not have recourse to the general formula. Their spectral classes and effective temperatures are known; determinations of parallax and absolute magnitude are available, and from these the diameters may be computed (when first directly observed). The masses of Sirius and Procyon are known, and those of the rest may be obtained from the mass-luminosity law. The adopted data are given in Table XII.[1]

TABLE XII

COMPUTED EXTENT OF STELLAR ATMOSPHERES

| | α Orionis | α Scorpii | α Boötis | γ Cygni | α Persei | α Can. Min. | α Can. Maj. |
|---|---|---|---|---|---|---|---|
| Sp..... | cM2 | cM1 | gK0 | cF8 | cF5 | dF4 | A2s |
| $T_e$..... | 3000° | 3050° | 4200° | 5800° | 6400° | 6800° | 10,000° |
| Bolom. $M$. | −4.9 | −5.6 | −0.8 | −3: | −2: | +3.0 | +1.1 |
| Diameter. | 250 | 450 | 27 | (39) | ( 20) | (1.9) | (2.5) |
| Mass..... | (32) | (50) | (9) | (12) | (8) | 1.1 | 2.4 |
| $g$..... | 0.0005 | 0.00025 | 0.012 | 0.008 | 0.02 | 0.31 | 0.38 |
| $Y_R$..... | 125 | 270 | 20 | 110 | 65 | 5.3 | 20 |
| $z$..... | 0.22 | 0.23 | 0.64 | 1.00 | 1.02 | 1.02 | 0.78 |
| $C^{-1/2}Y_G$ . | 8 | 12 | 3.7 | 10 | 8 | 2.4 | 5.7 |
| $C^{-1/2}p_G$.. | 0.004 | 0.003 | 0.045 | 0.08 | 0.16 | 0.76 | 2.2 |
| $zY_R$..... | 27 | 32 | 13 | 110 | 67 | 5.5 | 15 |
| $C$..... | 4 | 4 | 2 | 1 | 1 | 1 | 1 |
| $Y_G$..... | 16 | 24 | 5 | 10 | 8 | 2.5 | 6 |
| Obs. $T_e$... | 2900 | 3000 | 3850 | 5600 | 6700 | 7850 | 9900 |
| Obs. $Y_0$... | 100 | 100 | 20 | 80 | 6 | 0.6 | 0.05 |

Diameters, masses, etc., are given in terms of the sun's as unit. Those not derived from direct observation are given in parentheses. The absolute magnitudes of γ Cygni and α Persei, being uncertain, are marked with colons. The sun's effective temperature is taken as 6000°, and the values of $z$ are the visual luminous efficiencies, taking that at 6000° as unity. The lines headed $C^{-1/2}Y_G$, $zY_R$, $C^{-1/2}p_G$ give what should be the relative amounts of material in the reversing layer and in the chromosphere, and the pressure in the reversing layer, provided that the effect of changing ionization upon general opacity were negligible. The ionization factor $C$ may be estimated from Table XI, with the aid of the data for titanium—an element of

[1] The basal data are in general those adopted in chap. xxi of Russell, Dugan, and Stewart's *Astronomy*. The masses of α Orionis, α Scorpii, and α Boötis are increased, the first two to allow for the heat-index, and the last, on account of the well-known anomaly in its spectroscopic parallax (*op. cit.*, p. 875). The parallax of α Orionis is taken as 0".015 and its angular diameter as 0".035, according to values communicated by van Maanen and Pease as expressing the best available values.

intermediate ease of ionization. Only rough values can be or need be given. The values of $T_e$, the photospheric temperature, and $Y_0$, the extent of the atmosphere, as judged by its content of metallic vapors, are given at the bottom of the table. The temperatures agree excellently with those derived from evidence of other sorts, except that Procyon comes out too hot. The observed "extent" of the atmosphere exceeds the computed, for the cooler stars, and is very much less for Sirius.

This deviation is in the direction which would occur if the corrections for excitation potential which have been applied in calculating the abundance of the various elements were too great; but this explanation is untenable, for the ultimate lines, for which $E=0$ and no correction is necessary, give substantially the same result as the rest.

For the redder stars, the discrepancy is hardly alarming in the present state of both theory and observation. For Sirius, it is at first sight surprisingly great; but no account has so far been taken of second-stage ionization. A simple calculation shows that at a point in Sirius' atmosphere, with a temperature of 8700° and the same electron pressure as was assumed to exist on the sun (Table X), the percentage of doubly ionized atoms should be 99 for *Ba*, 92 for *Ca*, 80 for *Sc*, 50 for *Ti*, 20 for *Mg*, and 3 for *Fe* and *Si*. Even with a liberal allowance for higher pressure in Sirius, it is clear that the estimates of abundance given in Tables XI and XII should be much increased.

It follows also that the chromosphere of Sirius must be composed almost entirely of doubly ionized atoms (except for the permanent gases) and that the ordinary enhanced metallic lines, as well as the arc lines, should originate at low atmospheric levels, there and in other A stars. The enhanced lines would therefore be weaker in Sirius for *Ti*, which is chromospheric in the sun, than for *Fe*, and this is actually the case.

It is quite possible, however, that the metallic vapors in Sirius are diluted with some other gas which contributes relatively more than in the sun to the general opacity. The enormous strength of its lines suggests that hydrogen is responsible. In the upper part of Sirius' atmosphere, hydrogen should be strongly ionized, but toward the bottom the ionization should be rather small, and the

value of $C$ might thus be considerably increased. The relative strength of the arc lines in Sirius, which leads to the high values of the pressure in Table VIII, remains incompletely explained, but it may be that the assumptions on which our calibration is based become inaccurate for these very weak and rather broad lines.

## 11. BEHAVIOR OF HYDROGEN

The hydrogen lines are very strong in all the stars here studied. Direct estimates on the spectrocomparator gave intensities as follows:

| | α Orionis | α Scorpii | α Boötis | γ Cygni | Sun | α Persei | α Can. Min. |
|---|---|---|---|---|---|---|---|
| $H_\alpha$..... | 50 | 60 | 35 | 30 | 40 | 25 | 45 |
| $H_\beta$..... | 45 | 45 | 30 | 25 | 30 | 25 | 40 |
| $H_\gamma$..... | 25 | 25 | 20 | 25 | 20 | 25 | 30 |

In Sirius they are much too strong to estimate.

There appears, therefore, to be much more hydrogen, in the excited states, in the atmospheres of the red giants than in the sun's atmosphere. With the smallest plausible allowance for the effect of the excitation potential of 10.15 volts, the abundance of normal hydrogen must be thousands of times greater in Antares or Betelgeuse than in the sun.

The great intensity of the hydrogen lines in the cooler giant stars, especially in the $c$ stars, and its conspicuous dependence on absolute magnitude have long been a puzzle. Hydrogen, being light, is peculiarly subject to radiation pressure; and the greater strength of the latter, in comparison with gravity, in stars of great absolute brightness, may afford the explanation. As this, however, would lead us to anticipate stronger hydrogen lines in γ Cygni and α Persei than are observed, other factors may also be at work.

## 12. SUMMARY

The present paper should be regarded as the report of a reconnaissance in new territory. The principal results so far reached are as follows:

1. There appears to be a serious and widespread departure from the conditions of thermodynamic equilibrium in stellar atmospheres, which has the result that the relative number of atoms in excited

states, especially those of high energy, is much greater than is indicated by theory. The well-known behavior of the hydrogen lines appears to be a particular instance of the phenomenon.

2. An empirical method of correction for the effects of this anomaly has been developed, which leads to a new method of determining stellar temperatures. The temperatures thus found for stars of classes M to A are in excellent agreement with the accepted scale.

3. The anomaly does not appear to affect ionization, at least to at all the same extent as excitation.

4. The strength of the enhanced lines of *Fe*, *Ti*, etc., in the red giants suggests that they are produced largely in extensive chromospheres, supported by radiation pressure.

5. The amount of metallic vapor above equal areas of the photosphere in the cooler *c* stars ($\alpha$ Orionis, $\alpha$ Scorpii, $\gamma$ Cygni) appears to be of the order of one hundred times as great as in the sun. It is of the order of ten times as great in $\alpha$ Boötis and $\alpha$ Persei as in the sun. The amount in Procyon is comparable with that in the sun, while that in Sirius is smaller, but cannot be closely estimated, on account of higher ionization.

6. These results are in fair agreement with the existing theory of the production of a photosphere by general opacity in atmospheres of similar composition. It appears probable that in Sirius the metallic vapors are diluted by some other constituent (hydrogen?).

7. The solar spectrum, compared with the spectra of the brighter stars in general, is characterized by the relative weakness and notable sharpness of its lines. Procyon most closely resembles it in these respects, and it may turn out that these are general characteristics of dwarf stars (except in the latest types where the ultimate arc lines are very strong).

A very large amount of work can probably be done with profit in this field. When precise measures of line intensities take the place of rough estimates, and the study is extended to many more stars and to the lines of other elements, some of the preliminary results here presented will probably require revision. It is hoped to continue such work indefinitely at this Observatory.

CARNEGIE INSTITUTION OF WASHINGTON
MOUNT WILSON OBSERVATORY
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# PREDICTED LINES OF *Cr* II IN THE SPECTRA OF THE SUN AND OF α PERSEI[1]

By THEODORE DUNHAM, JR.,[2] and CHARLOTTE E. MOORE

ABSTRACT

*Basis of identification.*—A study of the multiplets in the *Cr* II spectrum, as derived from unpublished terms communicated by Dr. C. C. Kiess, indicates that many of the lines to be expected on the basis of theory have not yet been observed in the laboratory. The theoretical positions of these lines have been calculated, and a search for them has been made in the spectra of the sun and of α Persei.

As a result, 33 lines in the solar spectrum which were previously unidentified have been assigned to $Cr^+$; 19 less certain identifications have been indicated as $Cr^+$?. The best agreement is in the ultra-violet. Further, 33 predicted lines of *Cr* II have been identified in the spectrum of α Persei. When the stellar line is a blend, as is often the case, the contribution of *Cr* II has been estimated by taking account of the stellar intensity, wave-length, and all other possible identifications. In the green, several stellar lines of moderate intensity appear to be almost unblended and due primarily to predicted *Cr* II.

In making the identifications of *Cr* II each multiplet has been considered as a whole, attention being given to the theoretical intensities of the individual lines.

The spectrum of *Cr* has been measured by Exner and Haschek[3] and the behavior of the lines in the electric furnace has been studied by A. S. King.[4] The majority of the stronger lines known to belong to the spark spectrum of *Cr* are present in stellar spectra of types ranging from Ao to Go. In the spectra of super-giant stars such as α Persei and γ Cygni about thirty prominent lines are due to *Cr* II. In the solar spectrum these lines are clearly represented, the strongest being of intensity 5 on Rowland's scale. They are all definitely weakened in the sun-spot spectrum.

Dr. C. C. Kiess, of the Bureau of Standards, has very kindly communicated a number of unpublished terms in the spectrum of *Cr* II, thus making it possible to assign multiplet designations to most of the lines measured by Exner and Haschek. Many of the multiplets are incomplete, however, only the stronger members having been observed in the laboratory. Owing to the prominence

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 360.

[2] National Research Fellow.

[3] In Kayser, *Handbuch der Spectroscopie*, **5**, 337, 1910; **7**, 278, 1924.

[4] *Mt. Wilson Contr.*, Nos. 94 and 283; *Astrophysical Journal*, **41**, 167, 1915; **60**, 282, 1924.

of these stronger lines in stellar spectra of early type, a search for the fainter members has been made in the solar spectrum and in that of α Persei.

In the Revised Rowland Table[1] as many as possible of the solar lines have been identified by comparison with the available laboratory data for the various elements. Among the solar lines remaining unidentified, thirty-three have been attributed to *Cr* II, as their wave-lengths agree satisfactorily with those predicted by theory. In the ultra-violet the agreement is particularly striking, as almost every line required by theory coincides with a solar line. A number of lines have been identified as members of multiplets which have not yet been observed in the laboratory, but which, from the combinations of well-known terms in the *Cr* II spectrum, might be expected to occur. For lines in groups whose reality is doubtful or whose residuals (λ sun − λ arc) are large, the identifications of the solar lines have been indicated as $Cr^{+}$?; nineteen such lines have been included. A number of the predicted lines of *Cr* II are masked by those of other elements known to be present in the solar spectrum.

Further confirmation is obtained by a comparison with the spectrum of α Persei. A high-dispersion spectrogram taken at the coudé focus of the 100-inch reflector has been examined for the predicted lines between λ 3760 and λ 6600. Since the known lines of *Cr* II are much stronger on this plate than they are in the solar spectrum, the discovery of the predicted lines here should increase confidence in the corresponding solar identifications. Unfortunately the intrinsic width of all the lines is much greater in this spectrum than in that of the sun, and blends are therefore more frequent. This is particularly true in the blue and violet. Sometimes, however, the stellar line is definitely stronger than can be accounted for by the other known laboratory lines near the wave-length in question. In such cases it has been ascribed to a blend of *Cr* II and some other element. In identifying lines in the solar spectrum and in that of α Persei, each multiplet has been considered as a whole, attention being given to the theoretical intensities of the individual members as given by Russell.[2]

[1] *Carnegie Institution of Washington Publication*, No. 396, 1928.

[2] *Proceedings of the National Academy of Sciences*, 11, 322, 1925.

## TABLE I

PREDICTED LINES OF *Cr* II IN THE SPECTRA OF THE SUN AND OF α PERSEI

| PRED. λ I.A. | SUN | | | α PERSEI | | | | ν VAC.* | MULTIPLET DESIGNATION |
|---|---|---|---|---|---|---|---|---|---|
| | λ I.A. | Int. | Id. | λ I.A. | Int. | Blends | Contr. | | |
| 2976.72† | .697 | −2 | *Cr*+ | ..... | ..... | ..... | ..... | 33584.47 | $a^4P'_3-b^4D'_3$ |
| 3059.39 | .377 | 0 | *Cr*+ | ..... | ..... | ..... | ..... | 32676.89 | $a^2D_3-a^4D'_2$ |
| 3059.52† | .511 | 2 | −*Cr*+ | ..... | ..... | ..... | ..... | 32675.50 | $a^2D_2-a^4D'_2$ |
| 3336.13 | .129 | 0 | *Cr*+ | ..... | ..... | ..... | ..... | 29966.26 | $a^2D_2-a^4F_4$ |
| 3349.63 | .653 | −1 | *Cr*+ | ..... | ..... | ..... | ..... | 29845.32 | $a^2D_2-a^4F_3$ |
| 3353.13† | .130 | 2 | *Cr*+? | ..... | ..... | ..... | ..... | 29814.34 | $a^4D_4-a^6D'_5$ |
| 3467.11 | .136 | −2N | *Cr*+ | ..... | ..... | ..... | ..... | 28833.99 | $a^4D_3-a^6P_4$ |
| 3603.64 | .622 | 2 | −*Cr*+ | ..... | ..... | ..... | ..... | 27741.99 | $a^2D_3-b^4P_2$ |
| 3603.81† | .782 | 3 | −*Cr*+ | ..... | ..... | ..... | ..... | 27740.75 | $a^2D_2-b^4P_2$ |
| 3662.59 | .625 | −3N | *Cr*+? | ..... | ..... | ..... | ..... | 27295.03 | $a^4D_1-a^6F_1$ |
| 3665.47 | .438 | −3 | *Cr*+ | ..... | ..... | ..... | ..... | 27274.11 | $a^4D_2-a^6F_2$ |
| 3669.69 | .687 | 1 | −*Cr*+ | ..... | ..... | ..... | ..... | 27242.52 | $a^4D_3-a^6F_3$ |
| 3675.00 | .000 | −3 | *Cr*+ | ..... | ..... | ..... | ..... | 27203.17 | $a^4D_4-a^6F_4$ |
| 3688.02 | 7.987 | −3 | *Cr*+ | ..... | ..... | ..... | ..... | 27107.35 | $a^4D_3-a^6F_2$ |
| 3736.53 | .594 | −2 | *Cr*+? | ..... | ..... | ..... | ..... | 26754.79 | $b^4D_4-a^4F_4$ |
| 3755.14 | .136 | 0 | *Cr*+ *CN* | ..... | ..... | ..... | ..... | 26622.63 | $b^4D_3-a^4F_3$ |
| 3766.65 | .668 | 3 | *Fe* | .03 | 4 | *Fe, Zr*+ | C | (26541.26) | $b^4D_2-a^4F_2$ |
| 3767.23 | .206 | 8 | *Fe* | | | | C | (26537.22) | $b^4D_3-a^4F_2$ |
| 4030.37 | .350 | 0 | −*Sr* | .64 | 8 | *Mn* | C | (24804.61) | $b^4D_4-a^6D'_5$ |
| 4053.44 | .434 | 0 | *Cr*+ | .82 | 7nv | *Ti*+ | C+ | 24663.52 | $b^4D_2-b^4P'_2$ |
| 4054.10‡ | .078 | 0 | *Cr*+ | | | | C+ | 24659.57 | $b^4D_3-b^4P'_2$ |
| 4064.05 | .054 | 1 | *Cr*+ − | ..... | ..... | ..... | ..... | 24599.08 | $b^4D_3-a^6D'_4$ |
| 4072.63 | .514 | 2 | *Fe* | .61 | 2 | *Fe* | B−? | (24547.27) | $a^4P'_1-a^4D_2$ |
| 4075.66 | .708 | 0 | *Ce*+ | .87 | 4 | *Ce*+, *Sm*+? | C+ | (24529.01) | $b^4D_1-b^4P_2$ |
| 4076.87 | .884 | 1 | *Cr*+ | .73 | 5 | *Fe* | C+ | 24521.66 | $b^4D_2-b^4P_2$ |
| 4077.55 | .582 | 0N | *Cr*+ | .71 | 12 | *Sr*+ | C | 24517.45 | $b^4D_3-b^4P_2$ |
| 4087.63 | .607 | −1 | *Cr*+ | ..... | ..... | ..... | ..... | 24457.29 | $b^4D_1-b^4P_1$ |
| 4088.85 | .852 | −1 | *Ce*+ | .87 | 2 | *Fe, Ce*+ | B? | (24449.87) | $b^4D_2-b^4P_1$ |
| 4112.57 | .570 | −1Nd? | *Cr*+ | ..... | ..... | ..... | ..... | 24308.86 | $b^4D_2-a^4P_2$ |
| 4146.45§ | .500 | −1 | *Cr*+? | ..... | ..... | ..... | ..... | 24109.95 | $a^4P'_2-a^4D'_1$ |
| 4170.64 | .639 | −1 | *Cr*+ *CN*? | ..... | ..... | ..... | ..... | 23970.40 | $b^4D_1-a^4P_2$ |
| 4171.92 | .909 | 2 | *Fe, Ti*+ | .92 | 10 | *Ti*+ | C | (23963.07) | $b^4D_2-a^4P_2$ |
| 4172.62 | .590 | −1 | *Cr*+ | .71 | 4 | *Fe, Fe* | C+ | 23959.20 | $b^4D_3-a^4P_2$ |
| 4207.34 | .409 | 1N | *CN* | .20 | 4n | *Fe* | B− | (23761.34) | $a^4P'_2-a^4D'_3$ |
| 4215.78 | .813 | −2 | *CN* | .54 | 11 | *Sr*+ | C | (23713.75) | $b^4D_1-a^4P_1$ |
| 4217.08 | .062 | −1 | *Cr*+ | .27 | 1n | *Gd*+? | B? | 23706.54 | $b^4D_2-a^4P_1$ |
| 4229.82 | .776 | 3 | *CH Fe* | .70 | 3n | *Fe* | C | (23635.02) | $a^4P'_2-a^4D'_2$ |
| 4233.28 | .248 | −1 | *Cr*+ | .19 | 9 | *Fe*+ | C+ | 23615.88 | $a^4F'_4-a^4D'_4$ |
| 4279.00 | Abs. | ..... | ..... | .00? | 1 | *Mo*+? | A? | (23363.41) | $b^4D_1-a^6P_2$ |
| 4280.34 | .343 | −2 | *Ti*? | .52 | 2 | *Cr, Gd*+? | C+ | (23356.10) | $b^4D_2-a^6P_2$ |
| 4281.08 | .102 | 2 | *Mn* | .00 | 2n | *Mn* | B? | (23352.06) | $b^4D_3-a^6P_2$ |
| 4504.56 | .544 | −3 | *Cr*+ | .80 | 2 | −*Fe* | C+? | 22193.62 | $b^4D_4-a^6F_4$ |
| 4507.22 | .232 | 0 | *Cr*+ − | .19 | 1 | ..... | A? | 22180.37 | $b^4D_3-a^6F_4$ |
| 4542.82 | .847 | −3 | *Cr*+ | ..... | ..... | ..... | ..... | 22006.46 | $b^4D_4-a^6F_3$ |
| 4544.69 | .699 | 3 | *Ti* | .79 | 1 | *Cr, Ti* | C | (21997.56) | $b^4D_2-a^6F_3$ |
| 4558.84 | .779 | −2 | −*Cr*+? | .66 | 7 | *Cr*+ | C+ | 21929.57 | $b^4F'_3-a^4D'_4$ |
| 4571.30 | .300 | −2Nd? | *Cr*+ | .09 | 2 | *Mg* | C | 21869.51 | $b^4D_1-a^6F_2$ |
| 4572.83 | .871 | 0N | *Cr*+ ?− | .80 | 0 | ..... | A? | 21861.99 | $b^4D_2-a^6F_2$ |
| 4588.40 | .402 | −1N | *Cr*+ | .18 | 7 | *Cr*+ | C | 21788.00 | $b^4D_1-a^6F_1$ |
| 4589.89 | .955 | 3 | *Ti*+ | .96 | 8 | *Ti*+ | C+ | (21780.92) | $b^4F'_2-a^4D'_3$ |
| 4621.51 | .481 | −1 | *Cr*+? | ..... | ..... | ..... | ..... | 21632.05 | $a^4P'_1-a^4F_2$ |
| 4698.74 | .773 | 1 | *Ti* | .60 | 5n | *Cr, Ti* | C? | (21276.35) | $a^4P_2-a^4F_2$ |
| 4777.74 | .731 | −2 | *Nd*+? | .76 | 1 | *Nd*+? | A? | (20924.55) | $a^4P'_3-a^4F_4$ |
| 4805.18 | .102 | 3 | *Ti*+ | .14 | 9 | *Ti*+ | C? | (20805.08) | $a^4P'_2-a^4F_3$ |
| 4824.99 | .964 | −2 | *Cr*+? | .02 | 0 | ..... | A? | 20719.70 | $a^4P'_3-a^4F_2$ |
| 4876.53 | .486 | −1 | *Cr*+ | .47 | 8 | *Cr*+ | B− | 20500.84 | $a^4F'_4-a^4F_3$ |
| 5097.41 | Abs. | ..... | ..... | .04 | 4nd? | *Fe, Ni* | B−? | (19612.36) | $a^4P'_1-b^4P_2$ |
| 5116.15 | .192 | −3 | *Cr*+ | .26 | 0 | ..... | A? | 19540.37 | $a^4P'_1-b^4P_1$ |
| 5153.59 | Abs. | ..... | ..... | .36 | 4 | *Fe*− | B? | (19308.56) | $a^4P'_2-b^4P_3$ |
| 5210.97 | .943 | −3 | *Cr*+ | .61 | 2n | *Ti* | B | 19185.07 | $a^4P'_2-b^4P_1$ |
| 5232.48 | .517 | −2 | *Cr*+? | .88 | 8v | *Fe* | C+ | 19105.94 | $b^4F'_4-a^4F_5$ |

TABLE I—*Continued*

| Pred. λ I.A. | Sun | | | α Persei | | | | ν Vac. | Multiplet Designation |
|---|---|---|---|---|---|---|---|---|---|
| | λ I.A. | Int. | Id. | λ I.A. | Int. | Blends | Contr. | | |
| 5246.86 | .783 | −2 | *Cr*+? | .85 | 3n | *Fe* | B | 19054.02 | $a^4P'_1-a^4P_2$ |
| 5249.56 | .587 | −3 | *Nd*+ | .41 | 3 | *Nd*+? | A | (19043.93) | $a^4P'_2-a^4P_3$ |
| 5274.96 | .980 | 0 | *Cr*+?−*Fe* | .01 | 5 | *Cr, Fe* | A− | 18952.16 | $b^4F'_4-a^4F_4$ |
| 5279.89 | .880 | 0 | *Cr*+? | .02 | 4 | ..... | A | 18934.57 | $b^4F'_5-a^4F_4$ |
| 5280.11 | .071 | −1 | *Fe* | | | | | (18933.75) | $b^4F'_3-a^4F_4$ |
| 5318.49 | Abs. | ..... | ..... | .46 | 1 | *Sc*+ | A− | (18797.10) | $a^4P'_1-a^4P_1$ |
| 5346.08 | .092 | −1 | *Cr*+ | .92 | 5 | *Cr* | B | 18700.08 | $a^4P'_3-b^4P_2$ |
| 5368.09 | .122 | −3 | *Cr*+? | ..... | ..... | ..... | ..... | 18623.33 | $a^4F_2-b^4P_3$ |
| 5419.41 | .425 | −3 | *Cr*+? | ..... | ..... | ..... | ..... | 18447.01 | $a^4F'_3-b^4P_2$ |
| 5430.37 | .368 | −1 | *Cr*+? | ..... | ..... | ..... | ..... | 18409.85 | $a^4F'_2-b^4P_1$ |
| 5472.55 | Abs. | ..... | ..... | .53 | 0 | *Fe* | A− | (18267.96) | $b^4G_4-a^4F_4$ |
| 5707.91 | .922 | −3N | *Cr*+? | ..... | ..... | ..... | ..... | 17514.67 | $a^4F'_3-a^6P_4$ |
| 5742.81 | .815 | −3 | *Cr*+? | ..... | ..... | ..... | ..... | 17408.23 | $a^4F'_2-a^6P_3$ |
| 5926.19 | .205 | −3 | *Cr*+? | ..... | ..... | ..... | ..... | 16869.53 | $b^4F'_4-b^4P'_3$ |
| 5979.32 | .311 | −3 | *Cr*+? | ..... | ..... | ..... | ..... | 16719.73 | $b^4F'_2-b^4P_2$ |

* Wave-numbers in parentheses are predicted values. All others are derived from the solar wave-length in the second column.

† Observed in spark by Exner and Haschek; measures in arc discordant with predicted position.

‡ Measure by Exner and Haschek in spark at λ 4054.12 does not agree with solar wave-length.

§ *Cr* I observed at λ 4146.49 (2 IV).

‖ Line measured by Exner and Haschek in arc at λ 4232.87; in spark at λ 4233.24.

The strongest predicted lines in the stellar spectrum are members of the $a^4F'-a^4F$, $a^4P'-b^4P$, $b^4F'-a^4F$, and $a^4P'-a^4P$ groups in the green. A number of these lines are free from blends. Exner and Haschek measured a line at λ 4876.42 which appears to represent a blend of two members of the $a^4F'-a^4F$ group at λ 4876.39 and λ 4876.53. The pair is clearly resolved in the solar spectrum, both lines being much weakened in sun-spots. The stellar line is of intensity 8, a blend of both lines. The predicted line at λ 5232.48 ($b^4F'_4-a^4F_5$) is in the violet wing of the strong iron line at λ 5232.95. The stellar line is measured at λ 5232.88 and has a distinct wing on the violet side, indicating that the *Cr* II line makes a sensible contribution. This is confirmed by the presence of a solar line of intensity −2 at λ 5232.517.

The predicted lines as identified in the spectrum of the sun and of α Persei are listed in Table I. The first column contains the predicted wave-lengths of the *Cr* II lines; the second, third, and fourth columns give, respectively, the solar wave-lengths, intensities, and identifications; the fifth, sixth, and seventh columns give similar data for α Persei. In the eighth column the symbols *A*, *B*, and *C* indicate the extent of the contribution of *Cr* II to the line observed in α Persei;

*A* meaning that it predominates, *B* that it is comparable in importance with some other element, and *C* that it is masked. The ninth column gives the wave-numbers *in vacuo* of the solar lines, except that values in parentheses are predicted wave-numbers. The last column shows the multiplet designations.

The data listed in Table I form only a supplement to the work of Dr. Kiess, whose analysis of the *Cr* II spectrum has made this note possible. To him we wish to express our appreciation of his generous communication of material in advance of publication.

CARNEGIE INSTITUTION OF WASHINGTON
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# THE ORBITS OF THE SPECTROSCOPIC BINARIES Σ 674A AND Σ 1561B[1]

By ROSCOE F. SANFORD


## ABSTRACT

*The binaries Σ 674A and Σ 1561B* are components of Struve double stars which have proved to be spectroscopic binaries. The absolute magnitudes involved in the visual components are quite normal for their types. Σ 1561 has the fairly large annual proper motion, 0″.583.

*Spectroscopic elements.*—The components of Σ 674A are so nearly alike in magnitude and spectral class that the lines of both appear on its spectrograms. Circular orbits satisfy the radial velocities from the thirty-six spectrograms which have been obtained. The elements are: period, 3.4347 days; systemic velocity, −27.4 km/sec.; semi-amplitudes of velocity variation, 95.8 and 101.3 km/sec. for primary and secondary, respectively. It follows that $m_1 \sin^3 i = 1.4038$ ⊙ and $m_2 \sin^3 i = 1.3276$ ⊙. The agreement of the differences in the absolute and apparent magnitudes of A and B and the difference in their radial velocities suggests a physical connection of the two stars. There is, however, no certain change in position angle and distance since 1828.

Σ 1561B is a dwarf spectroscopic binary showing but one spectrum. Twenty-three radial velocities give an orbit for which the period is 23.5415 days; the systemic velocity, −13.8 km/sec.; the semi-amplitude of velocity variation for the primary, −25.8 km/sec.; eccentricity, 0.35; angle of periastron, 0°.00; and time of periastron passage, J.D. 2424303.000 G.M.T. The rather large eccentricity and the small semi-amplitude of velocity variation are noteworthy. The mean radial velocity of Σ 1561A from seventeen spectrograms is −18.2 km/sec. as compared to −13.8 km/sec. for Σ 1561B. This is probably close to its true velocity. The range in the individual velocities suggests, however, that A itself may be a spectroscopic binary. The radial velocities of A and B are consistent with motion in their visual orbits.

The absolute magnitude, radial velocity, and proper motion of the tenth-magnitude companion to Σ 1561AB indicate that it is not a part of the system AB.


This paper deals with the determination of the orbital elements of two spectroscopic binaries that are components of known visual double stars. Table I summarizes some general data in regard to them.

TABLE I

| Struve No. | Burnham No. | H.D. No. | App. Mag. | Sp. | α 1900 | δ 1900 | μ | Sp. Abs. Mag. |
|---|---|---|---|---|---|---|---|---|
| 674A.... | 2625A | 34335A | 6.8 | F5 | $5^h 11^m.6$ | +20° 1′ | ..... | 2.8± |
| 1561B.... | 5858B | 101177B | 8.2 | K4 | 11 33.5 | +45 40 | 0″.583 | 6.7 |

One-prism spectrographs equipped with 18-inch cameras have been used for the investigation, except for a few plates of the second

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 361.

star, for which shorter cameras were substituted. The plates taken with the 60-inch reflector are marked by a γ; those with the 100-inch, by a C. The radial velocities in all cases are means of at least two measures.

## Σ 674A

Burnham has numbered this visual binary 2625 in his *General Catalogue* and remarks that it appears unchanged over an interval of thirty-five years. Lewis,[1] however, says there is some slight evidence for a slow decrease in the distance, a point which a few good modern measures should have been able to settle.

Σ 674A was observed spectroscopically at this Observatory because its absolute magnitude would be of interest in case the suspicion of motion in the visual orbit were verified.[2] The first two spectrograms showed single-lined spectra that gave fairly accordant values of the radial velocity. The third plate, however, showed double lines, presumably from the spectra of two similar components in orbital motion. Subsequent observations left no doubt as to the binary character of Σ 674A. Thirty-six spectrograms were ultimately obtained between September, 1926, and February, 1928. Their distribution was such that the true period was not discovered until late, which accounts for a considerable number of spectrograms having been obtained at phases inappropriate for the separation of the two spectra. About two-thirds of the plates, however, yield velocities for both components. Their spectral types are closely alike and have been classified by the Mount Wilson observers as F2s. The Henry Draper classification is F5. Table II gives the observational data, which need no comment.

Numerous trials and adjustments finally gave $P = 3^{d}.4347$. Figure 1 shows the plot of the velocities of Table II when referred to a single epoch by means of this period. The velocities of the primary are denoted by small circles; those of the secondary by larger ones, while crosses represent velocities from unresolved spectra. It is apparent that the period can scarcely be in error as much as $0^{d}.001$, for such a modification would introduce inadmissible changes in the

[1] *Memoirs of the Royal Astronomical Society*, **56**, 140, 1906.

[2] Dr. Aitken has kindly communicated the latest measures of this visual double, and it is his opinion that there is no certain change since it was first measured in 1828.

assemblage of the velocities, which extend over one hundred and thirty orbital revolutions. It is further evident from Figure 1 that simple sine curves will adequately represent the observations and

TABLE II

OBSERVATIONS OF Σ 674A

| PLATE NO. | DATE | G.M.T. | PHASE | VELOCITY | | RESIDUALS | |
|---|---|---|---|---|---|---|---|
| | | | | Prim. | Sec. | Prim. | Sec. |
| | | | | km/sec. | km/sec. | km/sec. | km/sec. |
| γ 14511... | 1926 Sept. 18 | $0^h37^m$ | $0^d919$ | − 33.9 | ..... | ..... | ..... |
| 14593... | Oct. 17 | 0 22 | 2.430 | − 29.2 | ..... | ..... | ..... |
| 14696... | Nov. 14 | 22 56 | 0.459 | + 31.7 | − 94.4 | − 5 | + 1 |
| 14704... | Nov. 15 | 22 00 | 1.420 | −113.5 | + 58.8 | − 4 | 0 |
| 14717... | Nov. 17 | 22 19 | 3.433 | + 62.9 | −128.4 | − 6 | 0 |
| 14730... | Nov. 19 | 23 48 | 2.060 | − 95.9 | + 48.0 | + 9 | − 7 |
| 14737... | Nov. 20 | 21 23 | 2.959 | + 45.2 | − 99.9 | +11 | − 7 |
| C 4119... | Nov. 21 | 23 5 | 0.596 | − 27.2 | ..... | ..... | ..... |
| 4143... | Dec. 16 | 22 41 | 1.536 | −115.4 | + 66.7 | + 3 | − 2 |
| γ 14768... | Dec. 19 | 19 24 | 0.964 | − 34.8 | ..... | ..... | ..... |
| C 4152... | Dec. 20 | 15 59 | 1.822 | −118.0 | + 74.4 | + 3 | + 2 |
| 4158... | 1927 Jan. 9 | 20 28 | 1.401 | −110.0 | + 66.7 | − 2 | + 9 |
| γ 14776... | Jan. 12 | 15 15 | 0.748 | − 23.0 | ..... | ..... | ..... |
| 14780... | Jan. 12 | 20 35 | 0.970 | − 29.4 | ..... | ..... | ..... |
| 14782... | Jan. 13 | 15 47 | 1.771 | −131.4 | + 64.2 | − 9 | − 9 |
| C 4431... | Sept. 12 | 0 41 | 2.713 | − 28.6 | ..... | ..... | ..... |
| γ 15268... | Sept. 15 | 0 25 | 2.266 | − 76.5 | + 26.0 | + 2 | − 1 |
| 15287... | Oct. 9 | 23 7 | 3.169 | + 50.9 | −104.7 | − 7 | +12 |
| C 4465... | Oct. 11 | 1 5 | 0.817 | − 33.0 | ..... | ..... | ..... |
| 4470... | Oct. 12 | 0 55 | 1.810 | −122.0 | + 80.8 | 0 | + 8 |
| γ 15311... | Oct. 13 | 0 56 | 2.811 | + 17.4 | − 74.4 | + 5 | − 5 |
| 15316... | Oct. 13 | 20 30 | 0.191 | + 56.6 | −121.3 | − 6 | + 1 |
| 15373... | Nov. 3 | 22 36 | 0.671 | + 2.0 | − 71.2 | − 3 | −10 |
| 15426... | Dec. 6 | 16 14 | 2.493 | − 28.1 | ..... | ..... | ..... |
| 15431... | Dec. 6 | 23 23 | 2.791 | + 9.8 | − 61.5 | 0 | + 5 |
| C 4570... | Dec. 7 | 18 55 | 0.170 | + 68.8 | −129.3 | + 5 | − 5 |
| 4577... | Dec. 8 | 14 58 | 1.006 | − 35.4 | ..... | ..... | ..... |
| γ 15455... | Dec. 31 | 17 9 | 0.054 | + 69.0 | −126.9 | + 1 | + 1 |
| 15458... | Dec. 31 | 21 34 | 0.238 | + 66.0 | −123.6 | + 6 | − 4 |
| 15460... | 1928 Jan. 1 | 16 21 | 1.020 | − 33.0 | ..... | ..... | ..... |
| 15468... | Jan. 2 | 22 5 | 2.259 | − 76.5 | + 40.1 | + 3 | +12 |
| C 4603... | Jan. 3 | 15 23 | 2.980 | + 39.4 | − 86.4 | + 2 | + 9 |
| 4607... | Jan. 3 | 21 22 | 3.229 | + 62.4 | −124.5 | + 1 | − 3 |
| γ 15482... | Jan. 4 | 19 24 | 0.713 | − 29.6 | ..... | ..... | ..... |
| 15590... | Feb. 8 | 15 30 | 1.203 | − 90.0 | + 20.7 | − 6 | −12 |

that the orbits are therefore sensibly circular and fully described by the systemic velocity, the semi-amplitudes of velocity variation, and an epoch. It is assumed that the value for the period is definitive. The semi-amplitudes were obtained by an inspection of the plot of velocities in Figure 1. $T$, the epoch, is arbitrarily chosen as

the time when the primary has its greatest positive velocity. The velocity of the system, $\gamma$, was found by taking the mean value of the velocities from plates, with unresolved components. Table III indicates these preliminary elements.

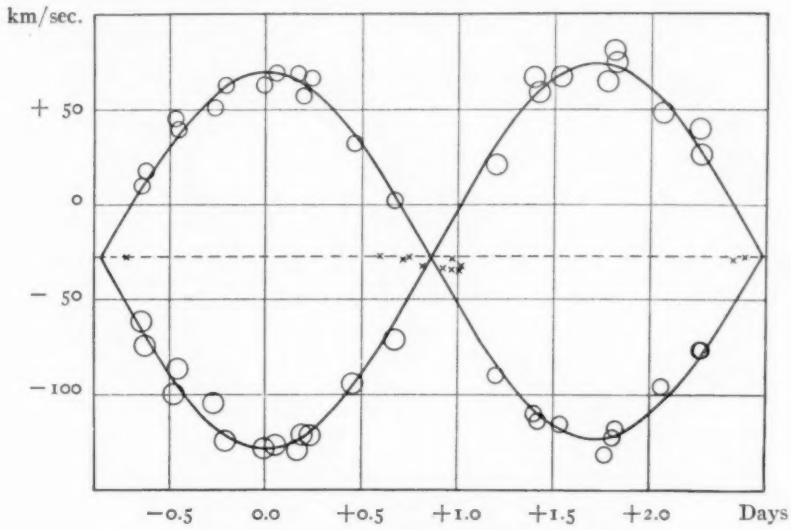



FIG. 1.—Radial-velocity curve of Σ 674A

Since the two spectra are so nearly alike, equal weights have been given to the velocities of the two components. In order to correct the four elements, $\gamma$, $K_1$, $K_2$, and $T$, it was therefore decided to derive

TABLE III

PRELIMINARY ELEMENTS

$P = 3.4347$ days (definitive)
$\gamma = -30$ km/sec.
$K_1 = 100$ km/sec.
$K_2 = 105$ km/sec.
$T$ = J.D. 2424837.960 G.M.T.

a single set of normal equations with the conditional equations from both primary and secondary, in accordance with a formula given by Harper.[1] The plates yielding only a single velocity were not, therefore, used for this solution. The residuals for the individual

[1] *Publications of the Dominion Astrophysical Observatory*, 1, 327, 1914.

velocities were computed with the aid of the preliminary elements. The least-squares solution then gave $d\gamma = +2.6$ km/sec., $dK_1 = -4.2$ km/sec., $dK_2 = -3.7$ km/sec., and $dT = -0^d.028$.

After making the foregoing corrections, the elements adopted for Σ 674A and the functions of mean distance and mass are as given in Table IV. The two velocity-curves from these elements are shown in Figure 1, together with a broken horizontal line for the velocity of the center of mass, $\gamma$. The probable error of a velocity of unit weight is $\pm 4.2$ km/sec.

The absolute magnitudes of the two components of Σ 674A given by their spectra are about 2.8. It is readily shown that the inclination, $i$, of the orbit plane needs differ only slightly from 90° in order to give masses which, with this value of the absolute magnitudes, harmonize well with Eddington's mass-luminosity relation.

TABLE IV

ADOPTED ELEMENTS

$P = 3.4347$ days
$\gamma = -27.4$ km/sec.
$K_1 = 95.8 \pm 1.09$ km/sec.
$K_2 = 101.3 \pm 1.09$ km/sec.
$T$ = J.D. 2424837.932 $\pm 0^d.005$ G.M.T.
$a_1 \sin i = 4{,}525{,}000$ km.
$a_2 \sin i = 4{,}784{,}000$ km
$m_1 \sin^3 i = 1.4038 \odot$
$m_2 \sin^3 i = 1.3276 \odot$

Only one spectrogram of the secondary star (vis. mag. 9.5) of the visual pair has so far been obtained. This gives the estimated spectral class K2, absolute magnitude 6, and radial velocity $-37$ km/sec. Although these values are consistent with the assumption of a physical connection between A and B, the evidence is hardly conclusive, because the foregoing values are derived from only one spectrogram of B, and also because there seems to have been no change in position angle and distance within a century.

## Σ 1561B

Burnham's *General Catalogue* gives the following remark concerning this visual double star:

No. 5858 = Σ 1561. The principal star has a large proper motion:

| | |
|---|---|
| Porter | 0".670 in 274°.0 |
| *A. G. Bonn* | 0.659 in 273.3 |

The motion of the companion is practically rectilinear and this may be a system of the 61 Cygni type. . . . . There is a distant 10$^m$ star, first measured by William Herschel and not in Struve, which does not belong to the system. . . . . If the distant star is fixed in space, which is likely to be the fact, the measures indicate that the proper motion of A, from meridian observations, is too large.

In his *Measures of Proper Motion Stars*, Burnham gives the value 0".59, which agrees with 0".583, the total proper motion in Boss's *Preliminary General Catalogue*. Apparently, therefore, the tenth-magnitude distant companion has, as Burnham inferred, a very small proper motion. Observations of this visual double, which were kindly placed at my disposal by Dr. Aitken, indicate a continued slow decrease in both position angle and distance.

The primary star is H.D. 101177 (appt. mag. 6.3, sp. type G0). Burnham estimates the secondary as 2 mag. fainter than the primary, while Adams[1] and his collaborators find F8 and K4, and 4.1 and 6.7, respectively, as the estimated spectral classes and the absolute magnitudes. The difference in the absolute magnitudes is consistent with the difference in the apparent magnitudes. Furthermore, it is evident that both components are normal stars of the main branch in Russell's diagram.

The measures of the first two spectrograms of Σ 1561B gave strong evidence that the radial velocity is variable, which later plates substantiated. It was then assumed to be a spectroscopic binary, and twenty-nine spectrograms were obtained between May 1919, and July 1927, for the determination of the orbit. Table V gives the data. Six spectrograms, as indicated in the table, were taken with cameras of short focus. The spectrum of only one star is registered on any of the plates.

The six velocities from low-dispersion spectrograms were not used in deriving the elements. The twenty-three remaining velocities can be satisfactorily assembled by using $P = 23^d.5415$, as has been done in Figure 2. It is evident from their arrangement that the orbit is an ellipse of considerable eccentricity. Elements deter-

[1] *Mt. Wilson Contr.*, No. 199; *Astrophysical Journal*, **53**, 61, 1921.

mined and corrected by methods suggested by Russell[1] have been adopted without revision by the method of least squares. These are given in Table VI, together with the functions of mean distance and mass.

TABLE V

OBSERVATIONS OF Σ 1561B

| Plate No. | Date | G.M.T. | Phase | Vel. | O−C |
|---|---|---|---|---|---|
| | | | | km/sec. | km/sec. |
| γ 8134 | 1919 May 8 | $18^h16^m$ | $21^d.207$ | + 2.2 | − 2 |
| C 276 | 1920 Feb. 5 | 21 28 | 11.839 | −26.6 | + 4 |
| 339 | Mar. 30 | 20 23 | 18.710 | −15.0 | 0 |
| 348 | Apr. 3 | 19 50 | 22.690 | +19.6 | + 2 |
| 402 | May 5 | 16 48 | 7.479 | −21.2 | + 4 |
| 1047 | 1921 June 11 | 17 47 | 9.314 | −24.9 | + 4 |
| 1659 | 1922 Apr. 10 | 19 44 | 6.356 | −22.0 | 0 |
| 2645 | 1924 Jan. 21 | 0 17 | 20.928 | + 0.2 | − 1 |
| 2686 | Feb. 17 | 21 19 | 1.718 | +22.8 | +12* |
| γ 12500 | Feb. 18 | 22 46 | 2.779 | −14.5 | −14* |
| 12504 | Feb. 19 | 22 44 | 3.777 | −12.0 | − 4* |
| 12506 | Feb. 20 | 19 59 | 4.655 | −16.0 | − 2* |
| C 3127 | 1925 Jan. 10 | 23 16 | 0.218 | +17.0 | − 4 |
| 3130 | Jan. 11 | 21 40 | 1.152 | +14.2 | − 1 |
| 3299 | May 29 | 18 16 | 21.305 | − 2.5 | − 8 |
| 3309 | June 4 | 18 24 | 3.769 | − 2.4 | + 6 |
| 3315 | June 6 | 15 50 | 5.660 | −14.2 | + 5 |
| γ 13446 | June 8 | 17 22 | 7.724 | −27.6 | − 1 |
| C 3337 | June 13 | 16 42 | 12.696 | −27.1 | + 3 |
| 3679 | 1926 Feb. 5 | 21 56 | 14.496 | −32.2 | − 4 |
| 3748 | Apr. 21 | 20 39 | 18.840 | −14.5 | 0 |
| 3766 | Apr. 25 | 16 3 | 22.632 | +15.8 | − 2 |
| 3828 | June 21 | 16 17 | 9.012 | −23.8 | + 5 |
| 3838 | June 22 | 16 4 | 10.005 | −23.0 | + 7 |
| γ 14856 | 1927 Mar. 15 | 21 39 | 17.281 | −26.1 | − 4* |
| C 4233 | Apr. 17 | 20 9 | 3.136 | − 2.8 | 0 |
| 4249 | May 9 | 15 42 | 1.049 | + 9.8 | − 4* |
| γ 14998 | May 21 | 16 57 | 13.461 | −43.2 | −13 |
| 15101 | July 11 | 16 23 | 17.355 | −30.3 | − 8 |

* Dispersion approximately one-half that for the remaining plates.

The velocity-curve which they define appears in Figure 2. The broken line shows the velocity of the system, γ. The residuals given in Table V have been read from this velocity-curve. Residuals are also included for the six plates of low dispersion, although they do not appear in the radial-velocity diagram.

Σ 1561B is, therefore, a dwarf spectroscopic binary which shows only the spectrum of its primary. The eccentricity is unusually

[1] *Astrophysical Journal*, 40, 282, 1914.

large. Beer[1] finds that dwarf spectroscopic binaries of about the period of Σ 1561B have a mean eccentricity of 0.18. The small semi-amplitude found here is affected, of course, by the value of the

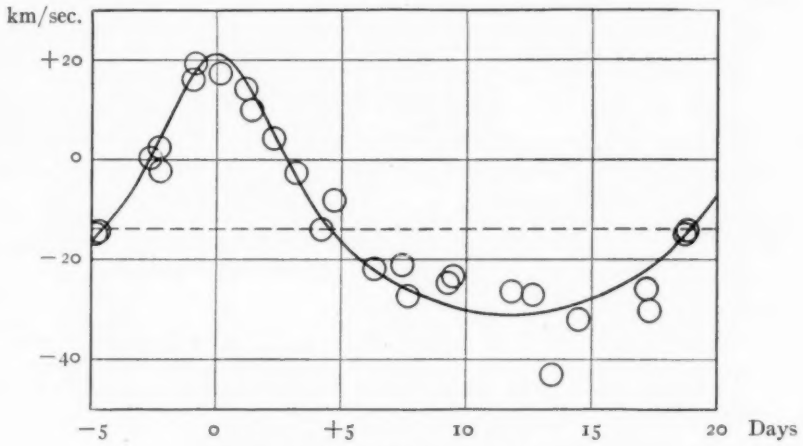



FIG. 2.—Radial-velocity curve of Σ 1561B

unknown inclination of the orbital plane. Nevertheless, it may well be that the orbital velocity is really small, for it appears that, on the average, small amplitudes are characteristic of dwarf spectroscopic

TABLE VI

ADOPTED ELEMENTS

$P = 23.5415$ days
$\gamma = -13.8$ km/sec.
$K_1 = 25.8$ km/sec.
$e = 0.35$
$\omega = 0.00$
$T =$ J.D. 2424303.000 G.M.T.
$a_1 \sin i = 7{,}806{,}400$ km
$\dfrac{m_2^3 \sin^3 i}{(m_1+m_2)^2} = 0.0343\ \odot$

binaries showing a single spectrum, a possible explanation of which lies in a small total mass and a small ratio of $m_2$ of the secondary to $m_1$ of the primary, as Beer[2] has pointed out.

[1] *Veröffentlichungen der Universitäts-Sternwarte zu Berlin-Babelsburg*, **5**, Sec. 6, Chart 7, 1927.

[2] *Ibid.*, p. 57.

The seventeen available determinations of the radial velocity of Σ 1561A give a mean of −18.2 km/sec. There is, however, evidence that the radial velocity of A is variable, for the individual velocities range from −8 to −31 km/sec. Hence the real difference between the true velocity of A and the systemic velocity of B (−13.8 km/sec.) will be subject to some uncertainty until the question of the variability of the radial velocity of A is cleared up. The indicated difference is, however, quite compatible with the orbital motions of the two stars of the visual pair about their common center of gravity. The similarity of the difference of the absolute magnitudes to the difference of apparent magnitudes and their known motion as a visual pair, together with undoubtedly nearly equal radial velocities, leaves no doubt of their physical association as a true double star.

One low-dispersion spectrogram of the tenth-magnitude companion to this visual double has been obtained. It appears to be a giant of spectral class G5 with a radial velocity of −32 km/sec. The very small proper motion, the giant characteristics of its spectrum, and its radial velocity all point to the fact that this star is not physically connected with the system Σ 1561AB.

CARNEGIE INSTITUTION OF WASHINGTON
MOUNT WILSON OBSERVATORY
March 1928

# THE ORBITS AND MASSES OF THE TWO COMPONENTS OF THE ECLIPSING VARIABLE RX HERCULIS[1]

By ROSCOE F. SANFORD

ABSTRACT

*Spectroscopic orbit.*—The elements of the circular orbits of the two components of RX Herculis, based on twelve spectrograms, are $\gamma = -24.9$ km/sec., $K_1 = 130.6$ km/sec., $K_2 = 146.5$ km/sec. The epoch and period given by Shapley in his formula for primary light-minimum seem to be quite satisfactory (Primary minimum = J.D. 2419658.5882 + $1^d.7785740E$ G.M.T.). It follows that $a_1 \sin i = 3{,}194{,}000$ km; $a_2 \sin i = 3{,}583{,}000$ km; $m_1 \sin^3 i = 2.078$ ⊙; and $m_2 \sin^3 i = 1.852$ ⊙.

*Photometric orbits.*—Two orbits which satisfy Shapley's light-curves are given in Table III. Solution II is probably to be preferred because it conforms more nearly to the relative brightness of the two components as furnished by the spectrograms. According to it, the light of the brighter star is 58.5 and that of the fainter 41.5 per cent of the total; the radii (distance of the centers = unity) are 0.200 and 0.180 for the brighter and fainter, respectively. The cosine of the angle of inclination is 0.065.

*Absolute dimensions.*—Combined spectroscopic and photometric data give 1.95 and 1.76 for the radii, and 0.280 and 0.344 for the densities of the brighter and fainter components, respectively, values for the sun being used as the unit in both cases.

*Mass-luminosity relation.*—The foregoing data and a temperature of 10,000° assumed from the spectral type (class A0) give absolute magnitudes but slightly less than are called for by Eddington's theory for the masses here obtained. The probable errors of the semi-amplitudes of velocity variation are large enough to cover the discrepancies. The discordance with Eddington's theory shown by Shapley's results derived from orbits based upon limited data is therefore removed.

RX Herculis[2] is an eclipsing variable star for which photometric orbits have been derived by Shapley.[3] Three determinations of the radial velocity of the two components, which had been obtained at the Yerkes Observatory, were used by Shapley to derive an approximation for the spectroscopic orbits. The phases for these velocities were calculated by his formula:

Primary minimum = J.D. 2419658.5882 + $1^d.7785740E$ G.M.T.

The result was circular orbits of identical dimensions in which $K_1 = K_2 = 106$ km/sec. (semi-amplitude of velocity variation); $\gamma = -18.5$ km/sec. (velocity of the system); $a_1 \sin i = a_2 \sin i =$

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 362.

[2] H.D. 170757; α, $18^h26^m.0$; δ, +12°33′ (1900).

[3] *Mt. Wilson Contr.*, No. 90; *Astrophysical Journal*, 40, 399, 1914.

2,590,000 km ($a$ is the radius of the circular orbits); and $m_1 \sin^3 i = m_2 \sin^3 i = 0.88\ \odot$.

Three velocities for each component of a spectroscopic binary are hardly enough to give trustworthy radial-velocity curves, even if the period and eccentricity are known from independent (photometric) data. In the case of RX Herculis the uncertainty is greatly increased because the two spectra are approximately of class Ao, and either spectrum by itself would be difficult of accurate measurement. When superposed, measurement is quite hopeless except at

TABLE I

SPECTROGRAPHIC OBSERVATIONS OF RX HERCULIS—MOUNT WILSON

| PLATE | DATE | G.M.T. | J.D. | PHASE | VELOCITY | | RESIDUAL | |
|---|---|---|---|---|---|---|---|---|
| | | | | | Prim. | Sec. | Prim. | Sec. |
| | | | | | km/sec. | km/sec. | km/sec. | km/sec. |
| γ 13515* | 1925 July 5 | $16^h13^m$ | 2424337.676 | $1^d438$ | + 84 | −158 | −13 | + 3 |
| C 3890† | 1926 July 22 | 19 23 | 2424719.808 | 1.177 | + 93 | −150 | + 7 | − 1 |
| 3895 | July 23 | 16 00 | 2424720.667 | 0.257 | −124 | +135 | + 4 | +45 |
| 3897 | July 23 | 18 58 | 2424720.790 | 0.380 | −164 | +109 | −12 | − 8 |
| 4291 | 1927 June 9 | 21 25 | 2425041.893 | 1.340 | +142 | −168 | +36 | + 3 |
| γ 15022 | June 11 | 18 33 | 2425043.773 | 1.441 | + 70 | −124 | −27 | +37 |
| 15103 | July 11 | 19 8 | 2425073.797 | 1.230 | + 90 | −180 | − 7 | −19 |
| 15108 | July 12 | 19 38 | 2425074.818 | 0.472 | −177 | +147 | −22 | +27 |
| 15184 | Aug. 13 | 19 3 | 2425106.794 | 0.434 | −152 | +105 | + 4 | −16 |
| 15264 | Sept. 14 | 16 40 | 2425138.694 | 0.319 | −148 | +107 | − 5 | 0 |
| 15269 | Sept. 15 | 16 28 | 2425139.686 | 1.311 | + 73 | −178 | −32 | − 7 |
| C 4460 | Oct. 10 | 15 30 | 2425164.646 | 1.371 | +105 | −171 | 0 | − 1 |

* Plates marked γ were obtained with the 60-inch reflector.

† Plates marked C were obtained with the 100-inch reflector.

phases for which their separation is near its maximum. Hence the spectroscopic orbit derived by Shapley is subject to considerable uncertainty. It seemed well worth while, therefore, to derive the spectroscopic orbits anew from additional spectrograms at appropriate phases.

This has been done with twelve spectrograms obtained with the 60- and 100-inch reflectors. In all cases the velocities are the mean of at least two measures. In ten out of twelve cases it was possible to pick out definitely the stronger spectrum, and the result shows consistently that the star to which it belongs is the one which is eclipsed at primary minimum. Table I gives the data of the twelve spectrograms.

The phases were computed from Shapley's formula for primary

minimum, already quoted. A plot was then made with the radial velocities as ordinates and the phases (in degrees) as abscissae. These points are shown as circles in Figure 1, which was used to derive approximate spectroscopic elements. Both the plotted velocities and Shapley's light-curve are well satisfied by the assumption of circular orbits. The velocities furnish a means of finding the velocity of the system and the amplitudes of velocity variation, while the light-curve gives the period and the epoch.

The preliminary values of the three elements from the velocity-curves were $\gamma = -25$ km/sec.; $K_1 = 131$ km/sec.; and $K_2 = 146$ km/sec. An ephemeris was then computed which furnished the

TABLE II

SPECTROSCOPIC ELEMENTS OF RX HERCULIS

$P = 1.7785740$ days
$T =$ J.D. 2419658.5882
$e = 0.00$ (assumed)
$K_1 = 130.6 \pm 4.0$ km/sec.
$K_2 = 146.5 \pm 5.3$ km/sec.
$\gamma = -24.9 \pm 4.0$ km/sec.
$a_1 \sin i = 3{,}194{,}000$ km
$a_2 \sin i = 3{,}583{,}000$ km
$m_1 \sin^3 i = 2.078\ \odot$
$m_2 \sin^3 i = 1.852\ \odot$

residuals O−C. A least-squares solution for corrections to the provisional values of the three elements gave $d\gamma = -0.10$ km/sec.; $dK_1 = -0.36$ km/sec.; and $dK_2 = +0.48$ km/sec., which are quite unexpectedly small. The values which result from these corrections, together with Shapley's epoch and period, have been adopted as the spectroscopic elements and are listed in Table II, which also gives the functions of mean distance and mass.

The residuals furnished by these elements are given in the last two columns of Table I. Although some of them are large, they are about what might be expected from the spectra involved. The probable error for a single plate of unit weight, upon which value the probable errors of $\gamma$, $K_1$, and $K_2$ are based, is $\pm 13.8$ km/sec.

Velocity-curves from these elements have been plotted at the top of Figure 1. Shapley's light-curve appears at the bottom. The

more massive star is the one which begins to approach immediately after primary minimum (phase = 0°). The three pairs of Yerkes[1] velocities have been plotted with squares. It is apparent that they can be satisfied only by smaller values of $K$, such as Shapley derived.

The adopted values of $K$ are 23 and 38 per cent greater, respectively, than Shapley's values, and lead to increases in $m_1 \sin^3 i$ and

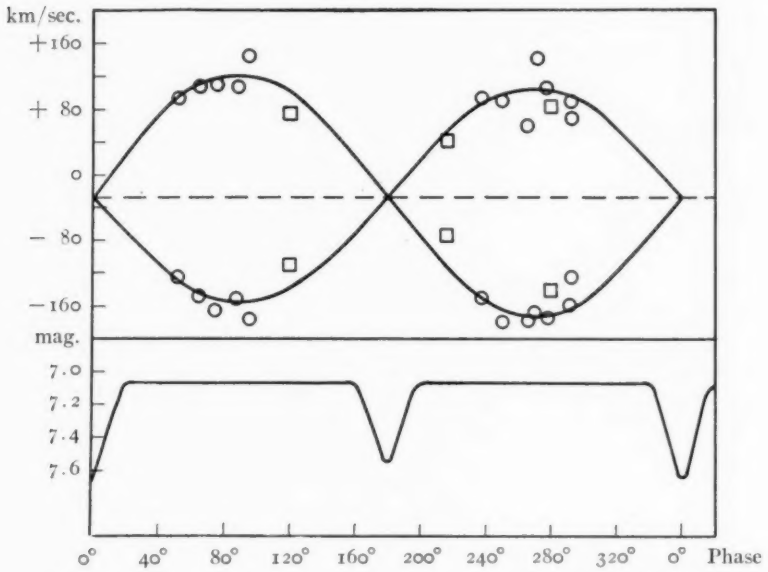



FIG. 1.—RX Herculis: Top, radial-velocity curves; primary has the smaller amplitude; circles are Mount Wilson radial velocities, squares are Yerkes values. Below, Shapley's light-curve.

$m_2 \sin^3 i$ of 136 and 110 per cent, respectively. It seems likely that the very poor quality of the spectral lines has resulted in measured separations which, by chance, are too small for all three of the Yerkes plates.

Shapley's data for RX Herculis present one of the few discordances with Eddington's mass-luminosity curve, the masses being smaller than demanded by the curve for the most reasonable assumptions of absolute magnitude. The new values of the masses agree much better with this curve. It is possible, however, to modify

[1] *Mt. Wilson Contr.*, No. 90; *Astrophysical Journal*, **40**, 399, 1914.

the photometric elements to conform more nearly to the evidence of the spectrograms, which show that the star eclipsed at primary minimum is noticeably brighter than the other, and this will be done before comparing the results with Eddington's theory.

I am greatly indebted to Professor Russell for the remaining computations, which involve two new solutions based on Shapley's light-curve, and on different assumptions for the relative brightness of the components. Table III gives the new photometric elements

TABLE III

| | | Solution I | Solution II | |
|---|---|---|---|---|
| **Photometric Elements** | | | | |
| Light of bright star | $L_b$ | 0.533 | 0.585 | |
| Light of faint star | $L_f$ | .467 | .415 | |
| Radius of bright star | $r_b$ | .190 | .200 | |
| Radius of faint star | $r_f$ | .190 | .180 | |
| Cosine of inclination | ..... | 0.070 | 0.065 | |
| **Photometric and Spectroscopic Elements Combined** | | | | |
| Radius of the bright star | | 1.86 | 1.95 | Sun is unity in all cases |
| Radius of the faint star | | 1.86 | 1.75 | |
| Density of bright star | | 0.324 | 0.280 | |
| Density of faint star | | 0.289 | 0.344 | |

and the linear dimensions derived from a combination of photometric and spectroscopic elements.

The spectral type Ao assigned to these stars indicates a surface temperature of 10,000°–11,000° C. The absolute magnitudes which correspond to the observed diameters and these estimates of the temperature are given by the equation on page 732 of Russell, Dugan, and Stewart's *Astronomy*. The absolute magnitudes corresponding to the observed masses can be obtained from Eddington's data. Table IV gives a comparison of these results. For $T = 10{,}000°$ the agreement is tolerable; for the higher temperature the residuals are rather large. All are systematically negative. It can be shown that an increase of $2\frac{1}{2}$ per cent in the values of $K$, which is less than their probable errors, would produce complete agreement for $T = 10{,}000°$. One reason that may be advanced for accepting the

lower temperature is that the adopted spectral type may be too early, because of the masking of weak metallic lines by the blending of the two superposed spectra. Solution II is probably preferable

TABLE IV

| | 11,000° | | | | 10,000° | | | |
|---|---|---|---|---|---|---|---|---|
| | Sol. I | | Sol. II | | Sol. I | | Sol. II | |
| | Brt. | Ft. | Brt. | Ft. | Brt. | Ft. | Brt. | Ft. |
| Absolute magnitude from temperature and radius. | 1.25 | 1.25 | 1.15 | 1.38 | 1.52 | 1.52 | 1.42 | 1.65 |
| Absolute magnitude from mass-luminosity law... | 1.62 | 2.01 | 1.62 | 2.01 | 1.56 | 1.95 | 1.56 | 1.95 |
| Residuals..... | −0.37 | −0.76 | −0.47 | −0.63 | −0.04 | −0.43 | −0.14 | −0.30 |

because it assigns a difference in brightness to the two stars agreeing with spectroscopic evidence. In any case, it is apparent that the new elements present no serious disagreement with Eddington's theory.

CARNEGIE INSTITUTION OF WASHINGTON
MOUNT WILSON OBSERVATORY
March 1928

# PHOTOGRAPHS OF VENUS[1]

By FRANK E. ROSS

ABSTRACT

*Photographs of Venus*, taken in *ultra-violet light* with the 60- and 100-inch reflectors of the Mount Wilson Observatory during the exceptionally favorable eastern elongation of June 1927, are reproduced in Plates I, II. They were taken on twenty-five nights, in as nearly an unbroken series as practicable.

*Details over the disk*, interpreted as *cloud formations*, are found to be *always present* on photographs taken with *ultra-violet light*. Photographs with *blue-violet* and with *blue* light show *very weak* details. *Red* and *infra-red* exposures disclose *no detail*. This behavior for light of different colors is interpreted as follows: The *outer atmospheric layer* is assumed to be composed of a *thin stratum of cirrus clouds*, while the *inner* atmosphere is supposed to be *exceedingly dense* and *yellowish*. *The details shown are atmospheric disturbances*, visible either as *dark* areas, or as regions of *enhanced brilliance*, depending upon the character of the disturbance.

*Bright clouds* generally appear near the *cusps*, which are presumed to be near the *poles*. The *dark areas* are generally in the form of *bands*, parallel to and favoring the assumed *equatorial* regions. The fact that these atmospheric disturbances are of the banded type, and the further fact that the atmospheric changes are found to be *rapid*, lead the writer to believe that the *period of rotation cannot be nearly so long as the orbital period*. On the other hand, conclusive *spectrographic data* will not admit of a short period. *A compromise period of about thirty days* seems to be the best that can be estimated from the existing data.

The literature on Venus, in so far as it bears on the problems suggested by the present series of photographs, is briefly reviewed.

Values of the *diameter* of Venus are deduced from the photographs in light of different colors. The negatives in *ultra-violet* light give a diameter *2 per cent greater* than those in *red* light. It is shown, however, that photographic determinations of diameter are subject to large systematic errors, to avoid which special precautions must be taken. The *value adopted* for the diameter is 17".00, corresponding to a linear diameter of 7651 miles, or 12313 km. The corresponding *mean density* is 0.919 of that of the earth, a value higher than previous determinations.

*Density curves* for a number of the negatives were recorded with the *registering microphotometer*. These were transformed into *intensity curves* showing the relative values of the light over the disk for various colors (Figs. 3, 4, 5). The variation is such that it seems likely that the *upper surface of the cloud layer is uniformly billowy*. Measures of the *relative values* of the light from the *dark* and the adjoining *bright* areas give a maximum difference of 24 per cent for the negatives in *ultra-violet* light, and of 4 per cent for those in *blue* light.

Through the courtesy of the director of the Mount Wilson Observatory, Dr. Walter S. Adams, the writer was accorded the privilege of using the 60-inch reflecting telescope for special photographic observations of the planets. The work is a continuation of that started in 1926, when particular attention was given to Mars.[2] The important feature of the present series of photographic studies is the use of filters of various spectral transmissions, the modern de-

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 363.

[2] *Astrophysical Journal*, **64**, 243, 1926.

velopment of which we owe to R. W. Wood, of Johns Hopkins University, and to William H. Wright, of the Lick Observatory.

On account of the interesting detail shown on the photographs of June 5 and 6 (Plate I), when the work was started, special attention has been given to Venus. Dr. Adams arranged that a long series of consecutive evenings should be available for daily photographs, a matter of great importance for a proper study of the rapid changes apparently taking place in the atmosphere of Venus, and of their bearing on the puzzling problem of the period of rotation and location of the axis. It is unfortunate that the time available for the photographs on any one day is limited, an unfavorable feature as compared with visual observations, which can be carried on during many consecutive hours of daylight. It is true, of course, that photographs in the longer waves, starting perhaps with the green, can be satisfactorily made in full daylight; but I have found that the photographs in ultra-violet cannot be exposed earlier than about ten minutes after sunset, and that those in blue-violet can begin only about sundown. The time available for the ultra-violet pictures, which are those most useful in showing the atmospheric phenomena, is, therefore, not greater than one hour. When the zenith distance exceeds 70°, the seeing becomes progressively worse, blotting out the delicate shades on the planetary surface. Since there is nearly always a complete change of cloud formation from day to day, a series of photographs extending over a period of six or eight hours would be exceedingly useful. In the case of the unusually strong markings, which doubtless can be photographed in yellow light, it is possible to obtain such a series. In view of the success of Mr. A. Rordame in securing photographs of a marking of this nature in full daylight (see p. 81 of this article), a field offering results of value is opened to amateurs possessing only very modest equipment.

In order to reduce the effect of changes in the form of the mirror, which are likely to occur near sundown, the 60-inch mirror was stopped to 52 inches. The 100-inch mirror was always used at full aperture.

An enlarging camera belonging to the Yerkes Observatory was used (Fig. 1). This is of special design, enabling the observer to

guide on the planet itself during the comparatively long exposures which are sometimes necessary, as, for example, in the case of Jupiter, photographed with an ultra-violet screen, when the exposures were of two-minute duration. (With the same screen the exposures on Venus were from two to three seconds.) The enlarging lens is of the negative type, of about 6 inches focal length. The filters were placed 2 inches in front of the focal plane. About an inch in front of the filter a very thin piece of unsilvered optical glass, of good

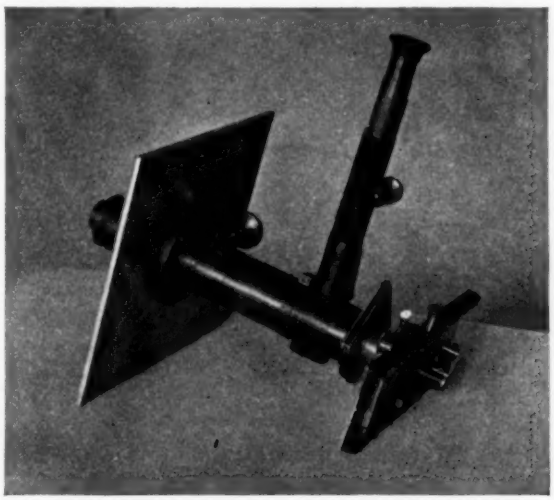


FIG. 1.—Enlarging camera

surface, was set at an angle of 45° to the axis of the instrument. This formed two images of the planet in the focus of an eyepiece of very low power, in which threads were placed for guiding. The double image, formed by the two reflecting surfaces of the inclined plate, is a necessary evil, but not unduly troublesome. This arrangement proved exceedingly useful, for the conditions of seeing and steadiness could always be judged to a nicety before an exposure was made. The guiding, necessary on exposures longer than about three seconds, was by screws operating a double-slide baseplate, to which the camera was attached. An iris shutter was placed directly in front of the plate. It must be noted that the zenith distance of Venus was considerable, and, consequently, that the un-

steadiness usually was very great, which made guiding a necessity. Five exposures were made on each plate. Focusing was judged on a ground glass, and also by the eye lens, as a check. Fine focusing was effected by means of a rack and pinion, which moved the enlarging camera as a whole.

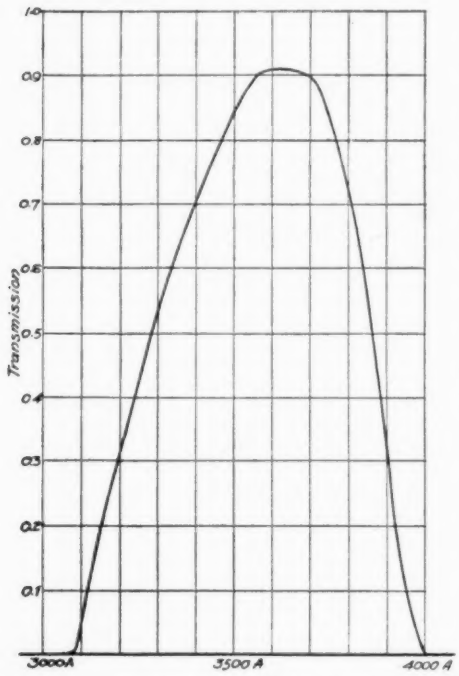



FIG. 2.—Spectral transmission of ultra-violet filter No. 18A.

*Filters.*—The new ultra-violet filter, Wratten 18A, manufactured by the Eastman Kodak Company, was used for the ultra-violet pictures. This proved many times faster than the No. 18 used in 1926 for Mars. Its transmission curve was determined by Dr. E. Pettit, and is shown in Figure 2. For the blue pictures, Wratten C filter was used. This transmits from 4000 to 5000 A, with a maximum at 4700 A. A glass filter by Schott, F 4512, cutting out everything shorter than about 6000 A, was used, both for the red and the infra-red photographs.

*Plates and development.*—For the ultra-violet, blue-violet (no filter), and blue photographs, Cramer Contrast plates were used. These are fine grained and of excellent quality. For the red photographs, the Ilford Process Panchromatic plate was employed. For the infra-red, the Eastman Kryptocyanin plate was used. These have a maximum of sensitivity at about 7700 A, with no sensitivity in the visible red. On account of the great range in the intensity of the light coming from different portions of the disk, it is necessary to employ a soft-working developer. Eastman Nepera developer, diluted 1 to 8, was found to answer the purpose very well. The tem-

perature, 70° F., was carefully controlled. Development time varied from one to four minutes, the range in exposure time being sufficient to allow considerable freedom in degree of development. Standard squares were impressed on many of the plates, the appropriate filter being used in each case. It is accordingly possible to measure the relative light-intensities over the disk of the planet (p. 90).

The program adopted was as follows: About thirty minutes before sundown, the planet was examined visually with a power of 240, through various filters, at the Cassegrain focus of the mirror, the enlarging camera being removed. The camera was then replaced, and one plate each of infra-red, red, blue, and blue-violet, in turn, was exposed before sundown. Five exposures were made on each plate. The ultra-violet exposures were then begun, and continued in rapid succession as long as the seeing, which progressively grew worse with diminishing altitude, warranted exposing.

*Scale value.*—Since it was planned to measure diameters, it was important to know the value of the scale. This was determined by exposures on the following wide double stars:

| | Separation | |
|---|---|---|
| | Yerkes | Burnham |
| $\beta$ G.C. 8136 | 20".60 | 20".65 |
| 6887 | 25.56 | 25.52 |
| 10012 | 43.20 | 43.18 |

The separations marked "Yerkes" were used, which agree remarkably well with the values given by Burham. They were determined by exposures made by Dr. G. W. Moffitt with the 40-inch refractor, which were measured and reduced by the writer. With the magnification usually employed, the equivalent focal length of the combination 60-inch mirror plus enlarging camera was found to be 238.0 feet (72.5 m); of the 100-inch mirror plus camera, 370.3 feet (112.8 m).

## THE PHOTOGRAPHS

In any series of photographs of the planets taken with light of various wave-lengths, it is to be expected that the long-wave, or red, photographs will penetrate the outer atmosphere of the planet and disclose features on the surface itself, provided the atmosphere is not too dense. Since the visible surface of Venus is doubtless atmospheric, there is a possibility that infra-red light may be able

to penetrate this atmosphere and reveal details of the surface. It was therefore a matter of some surprise to find that the relative efficiency of ultra-violet and infra-red light in showing detail on the visible disk was reversed: The ultra-violet negatives gave considerable detail, while the infra-red, on all occasions, showed nothing. At the time these photographs were made, I was not aware that Professor William H. Wright, in 1924, had made photographs of Venus, with a similar expectancy:

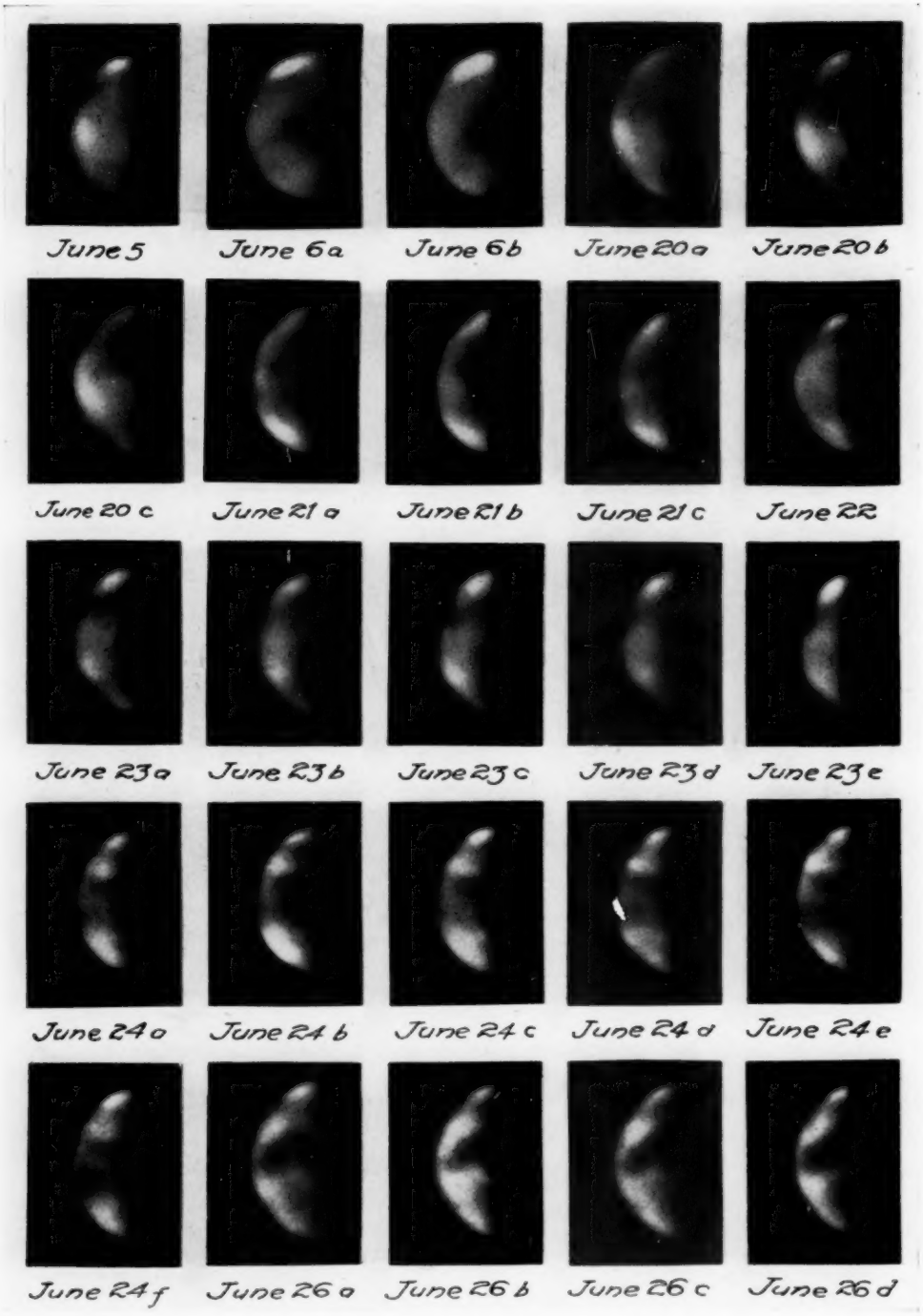
> While observing Mars during the opposition of 1924, by light of different colors, a number of photographs of Venus were secured for the purpose of determining whether infra-red light, which had been found useful in delineating the surface features of the former of these planets, would prove equally effective in penetrating the atmosphere of the latter, and in depicting whatever might lie beneath it.[1]

A number of the negatives taken with ultra-violet light are reproduced (Plates I and II). South is above, following the usual practice. For those days on which a great deal of detail is present, a number of exposures, taken at intervals of from five to ten minutes, are reproduced, as they are useful in studying possible changes, and, moreover, are convincing as to the reality of the detail. I am greatly indebted to Ferdinand Ellerman for the excellence of the enlarged prints from which the plates were prepared. On account of the great variation in the intensity of the light over the disk, and, in particular, on account of its weakness throughout the extended region adjoining the terminator, an enlarged transfer of the negative to a print is an exceedingly difficult matter, as anyone who tries it will know. Mr. Ellerman is to be congratulated on his success, and on his very beautiful prints. The details connected with each photograph shown in Plates I and II are collected in Table I. It will be noted that some of the negatives were made with the 100-inch mirror. While this gave a much greater scale, it is not felt that the negatives show any superiority over those obtained with the 60-inch mirror. In view of the severity of the observing conditions, this is not surprising.

In addition to negatives taken with ultra-violet and infra-red light, exposures were made each night with blue-violet light,

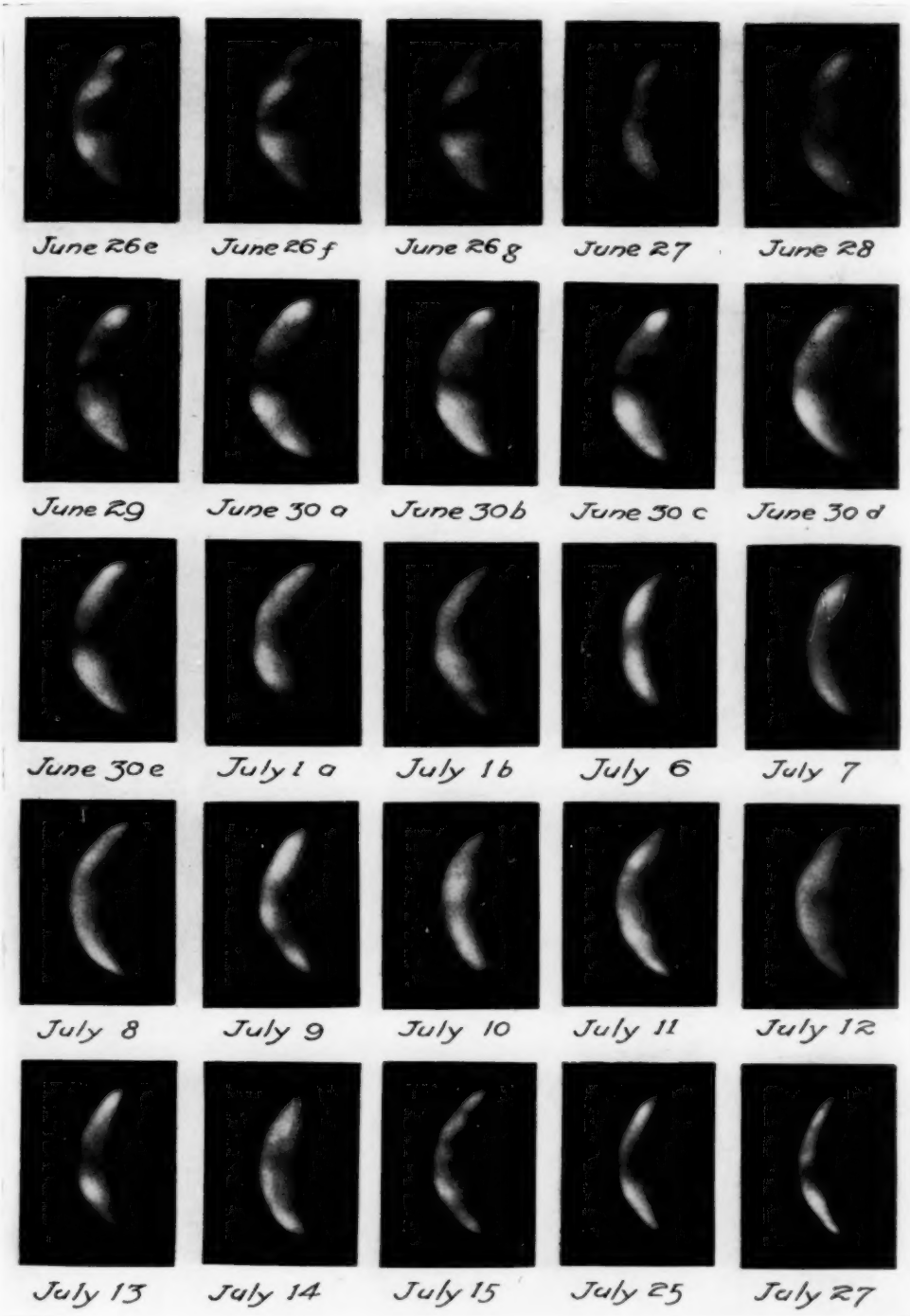
[1] *Publications of the Astronomical Society of the Pacific*, **39**, 220, 1927.

PLATE I




PHOTOGRAPHS OF VENUS, IN ULTRA-VIOLET LIGHT, TAKEN WITH 60-INCH REFLECTOR, JUNE 1927

PLATE II




UNIV. OF MICH.

PHOTOGRAPHS OF VENUS, IN ULTRA-VIOLET LIGHT, TAKEN WITH 60-INCH AND 100-INCH REFLECTORS, JUNE–JULY 1927

## TABLE I

### Data on Plates

| No. | Date | Pac. S.T. | U.T. | Seeing | Mirror | Diam. of Image | Hour Angle | Zenith Dist. |
|---|---|---|---|---|---|---|---|---|
| | 1927 | | | | Inches | mm | | |
| 1 | June 5 | $7^h17^m$ | $6^d3^h17^m$ | 1 | 60 | 6.4 | $4^m17^m$ | 56°.7 |
| 2 | 6 | 7 18 | 7 3 18 | 1 | 60 | 6.5 | 4 18 | 57.0 |
| 3 | 6 | 7 32 | 7 3 32 | 1 | 60 | 6.5 | 4 32 | 59.8 |
| 4 | 20 | 7 25 | 21 3 25 | 1 | 60 | 7.4 | 4 19 | 59.2 |
| 5 | 20 | 7 39 | 21 3 39 | 1 | 60 | 7.4 | 4 33 | 61.9 |
| 6 | 20 | 7 57 | 21 3 57 | 1 | 60 | 7.4 | 4 51 | 65.7 |
| 7 | 21 | 7 26 | 22 3 26 | 4 | 60 | 7.4 | 4 20 | 59.5 |
| 8 | 21 | 7 40 | 22 3 40 | 4 | 60 | 7.4 | 4 34 | 62.4 |
| 9 | 21 | 7 52 | 22 3 52 | 4 | 60 | 7.4 | 4 46 | 64.9 |
| 10 | 22 | 7 18 | 23 3 18 | 1 | 60 | 7.5 | 4 12 | 58.0 |
| 11 | 23 | 7 21 | 24 3 21 | 4 | 60 | 7.5 | 4 15 | 58.8 |
| 12 | 23 | 7 36 | 24 3 36 | 4 | 60 | 7.5 | 4 30 | 61.9 |
| 13 | 23 | 7 48 | 24 3 48 | 4 | 60 | 7.5 | 4 42 | 64.3 |
| 14 | 23 | 7 57 | 24 3 57 | 4 | 60 | 7.5 | 4 51 | 66.3 |
| 15 | 23 | 8 2 | 24 4 2 | 4 | 60 | 7.5 | 4 56 | 67.3 |
| 16 | 24 | 7 21 | 25 3 21 | 3 | 60 | 7.7 | 4 15 | 59 0 |
| 17 | 24 | 7 32 | 25 3 32 | 3 | 60 | 7.7 | 4 26 | 61.2 |
| 18 | 24 | 7 38 | 25 3 38 | 3 | 60 | 7.7 | 4 32 | 62.5 |
| 19 | 24 | 7 46 | 25 3 46 | 3 | 60 | 7.7 | 4 40 | 64.1 |
| 20 | 24 | 7 57 | 25 3 57 | 3 | 60 | 7.7 | 4 51 | 66.4 |
| 21 | 24 | 8 5 | 25 4 5 | 3 | 60 | 7.7 | 4 59 | 68.1 |
| 22 | 26 | 7 17 | 27 3 17 | 4 | 60 | 8.0 | 4 11 | 58.7 |
| 23 | 26 | 7 29 | 27 3 29 | 4 | 60 | 8.0 | 4 23 | 61.1 |
| 24 | 26 | 7 36 | 27 3 36 | 4 | 60 | 8.0 | 4 30 | 62.7 |
| 25 | 26 | 7 40 | 27 3 40 | 4 | 60 | 8.0 | 4 34 | 63.4 |
| 26 | 26 | 7 50 | 27 3 50 | 4 | 60 | 8.0 | 4 44 | 65.3 |
| 27 | 26 | 8 1 | 27 4 1 | 4 | 60 | 8.0 | 4 55 | 67.7 |
| 28 | 26 | 8 5 | 27 4 5 | 4 | 60 | 8.0 | 4 59 | 68.5 |
| 29 | 27 | 7 37 | 28 3 37 | 1 | 60 | 8.1 | 4 31 | 62.9 |
| 30 | 28 | 7 31 | 29 3 31 | 1 | 60 | 8.1 | 4 25 | 61.7 |
| 31 | 29 | 7 25 | 30 3 25 | 1 | 60 | 8.2 | 4 19 | 60.8 |
| 32 | 30 | 7 20 | 1 3 20 | 4 | 60 | 8.3 | 4 14 | 60.0 |
| 33 | 30 | 7 27 | 1 3 27 | 4 | 60 | 8.3 | 4 21 | 61.3 |
| 34 | 30 | 7 30 | 1 3 30 | 4 | 60 | 8.3 | 4 24 | 62.0 |
| 35 | 30 | 7 42 | 1 3 42 | 4 | 60 | 8.3 | 4 36 | 64.5 |
| 36 | 30 | 7 54 | 1 3 54 | 4 | 60 | 8.3 | 4 48 | 67.0 |
| 37 | July 1 | 7 24 | 2 3 24 | 3 | 60 | 8.4 | 4 19 | 61.2 |
| 38 | 1 | 7 44 | 2 3 44 | 3 | 60 | 8.4 | 4 39 | 65.3 |
| 39 | 6 | 8 5 | 7 4 5 | 3 | 100* | 5.1 | 5 2 | 71.1 |
| 40 | 7 | 7 40 | 8 3 40 | 3 | 100 | 15.0 | 4 37 | 66.1 |
| 41 | 8 | 7 41 | 9 3 41 | 5 | 60 | 9.1 | 4 39 | 66.9 |
| 42 | 9 | 7 42 | 10 3 42 | 6 | 100 | 15.1 | 4 41 | 67.4 |
| 43 | 10 | 8 0 | 11 4 0 | 5 | 100* | 5.3 | 4 59 | 71.5 |
| 44 | 11 | 8 1 | 12 4 1 | 6 | 100 | 15.2 | 5 1 | 71.9 |
| 45 | 12 | 7 27 | 13 3 27 | 2 | 100 | 15.4 | 4 28 | 65.4 |
| 46 | 13 | 7 47 | 14 3 47 | 3 | 100 | 15.7 | 4 49 | 69.9 |
| 47 | 14 | 7 46 | 15 3 46 | 4 | 100 | 15.8 | 4 49 | 70.1 |
| 48 | 15 | 7 25 | 16 3 25 | 3 | 60 | 9.5 | 4 29 | 66.3 |
| 49 | 25 | 7 24 | 26 3 24 | 3 | 60 | 11.2 | 4 40 | 70.1 |
| 50 | 27 | 7 23 | 28 3 23 | 5 | 60 | 11.4 | 4 43 | 72.1 |

* Without enlarging camera. Seeing is on an ascending scale from 1 (poor but usable) to 10 (perfect).

blue light, and red light. No details whatever are visible on the red or the infra-red negatives. Unscreened plates and plates taken with the blue filter on the dates of the strongest markings show some details however; the markings on those taken without filter are the stronger, but greatly inferior in quality and intensity to those observed in the ultra-violet. There is thus a continuous transition in degree of visibility, of both the dark and the bright markings, as we pass from short to long waves. A similar phenomenon is observed with Jupiter, photographs with blue-violet light showing greater contrast than those taken with red light. The bearing of this will be discussed in a following paragraph.

*Description of the photographs.*—The various features of the photographs reproduced on Plates I and II are brought out so clearly that the reader can follow for himself the story of the atmosphere of Venus during the interval covered by these observations. Those of interest and significance will be indicated in the text which follows, as they become necessary to the development of the subject. Here only a few miscellaneous matters will be considered.

It will be noticed that the prints show irregular variations of phase, even on the same day. These are due to slight differences in the intensity of the original negatives and to the difficulty of "holding" the true terminator while making the enlarged prints with the enhanced contrast necessary to bring out properly the surface detail. In fact, unless great care is taken, as much as one-half of the disk may be lost entirely in the process of reproduction.

Photographs of the same date show, in some cases, slight differences in detail which call for comment. These do not represent changes on Venus, but doubtless only differences in the negatives due to changes in seeing, in exposure time, and in other factors. It is to be remembered that the differences of density are multiplied many times in the reproductions, so that small differences in the original negative appear conspicuous in the engraving.

The striking irregularity in outline of the strongly illuminated limb of the planet is worthy of remark. It adds its testimony to the lack of homogeneity of the atmosphere and to the existence of dark and bright clouds. A dark cloud at the limb is evidenced by a depression, a bright one by a bulge. Many of each kind are seen on the plates.

*Visibility of the markings on the photographic images.*—Certain phenomena connected with the visibility of the markings on the negatives and on the reproductions are of interest. The average diameter of the image of Venus, in the present series, is about 8 mm. With normal vision, the markings are best seen without magnification. Even with a magnifier of power 2, the visibility becomes less. This cannot be a matter of contrast, which is unchanged by magnification. The true explanation is of interest and of importance in connection with the reproduction of the photographs. While part of the decreased visibility may be due to the grain of the negative, this is not the most important factor, which I believe to be the sharpness of outline of the markings. On account of the small differences of density involved, the sharpness of any given marking, defined as the rate of change of density as the boundary of the marking is crossed, must be very low. Now, as sharpness is inversely proportional to the magnification, the visibility must also be inversely proportional to magnification. This principle finds direct application to the reproduction of the photographs, and shows why enhanced contrast is necessary in order to make the markings as visible on enlarged prints as they are on the original negatives. In the present case, the enlargements range from two to eight times, the average being about four. If the original negatives are examined under a power of 4, the markings practically disappear. From this it may be concluded that a reproduction with fourfold enlargement would fail to show what is on the negatives. If, however, the contrasts are also increased four times, the sharpness will be maintained at its original value, and the markings will remain visible. This principle has been recognized in the process of reproduction of the present series. Since the contrasts between dark and light areas are much greater than they are in the originals, one should not judge the relative value of intensity of light from the prints.

The principle advanced in the preceding paragraph also finds direct application to the telescopic examination of the disk of Venus and some of the other planets. As in the case of the negative, we here have delicate shades of intensity, with a very low factor of sharpness. A decrease of the power of the telescope increases the sharpness, and, accordingly, the visibility in direct proportion.

There is, of course, a limit, depending upon the optics of the eye and its powers of discrimination. A great many factors are involved, which need not be discussed here; but there seems to be no doubt of the importance of sharpness in determining the power or ability of the eye to detect fine differences of shade, which do not have the definite boundaries which distinguish, for example, the canal-like markings of Mars, but which merge gradually into one another, as one would expect of markings of an atmospheric origin.

## INTERPRETATION OF THE PHOTOGRAPHS

Granted that Venus has an atmosphere which hides the surface from visual observation, it is normally to be expected that photographs taken with infra-red light, which would perhaps penetrate the atmosphere, might disclose surface features, just as during the great war photographers were able to take pictures of the earth's surface from points several miles above it by the use of infra-red light. Pictures of Mars were made in this way by William H. Wright in 1924. He found that the infra-red pictures penetrated through to the surface of the planet, while those taken in ultra-violet were practically stopped in the upper atmosphere. If the atmosphere of Mars were denser than it is, the effect would be more pronounced In that case visual observations would become difficult and infra-red pictures increasingly valuable If a planetary atmosphere such as exists on the earth is assumed, and one were photographing it from an outside point, clouds would be photographed more readily with infra-red than with shorter waves But, in photographing Venus, this does not appear to hold, the short waves now being the useful ones. What is the change in conditions which brings this about? In the case of the earth the difficulty in photographing with short waves is that the air strongly reflects violet and ultra-violet light. If we imagine the air mass decreased, the difficulty becomes less. When it has been decreased hypothetically to a certain point, photography of the clouds becomes possible with the short waves. But even this assumed condition does not match the results obtained for Venus, for a reduction of the air mass should not affect photography in the infra-red, except in special cases. The conditions on Venus evidently form a special case. Here we get no de-

tail in the infra-red, and, at times, a great deal in the ultra-violet. The following solution suggests itself to the writer. With more data, other explanations could doubtless be advanced.

If the surface of Venus is yellow or reddish, as in the case of Mars, but without detail, and the atmospheric mass is small, and, moreover, if water vapor is assumed to be present, although not necessarily in large amounts, then a series of pictures from above, in different colors, would agree with what has been found; thus ultra-violet pictures would be interpreted as showing clouds, of varying degrees of intensity, the dark patches being the actual surface of the planet showing through the occasional breaks in the cloud mantle overlying the planet. No detail would appear in the red or infra-red, since, on the assumption of a yellow or red surface of uniform structure, we have what may be called a fogging background, blotting out the delicate detail lying in front, assumed to be cloud masses of varying degree of tenuity. The ultra-violet pictures show this detail because the fogging background is absent, inasmuch as the planet's surface is assumed to reflect only light of the longer wave-lengths. If the mass of air were considerable, the ultra-violet light would become impotent, on account of the selectively scattered light.

These assumed conditions, however, are perhaps inadmissible. Other considerations, which will be outlined in another paragraph, require an atmospheric covering of considerable mass. It is easy to adapt the foregoing line of reasoning, which was advanced to explain the difference between the pictures taken with short and with long waves, to this new condition. It is only necessary to transfer the optical properties assumed for the surface of the planet to the top of an atmospheric shell, as many miles as we please above the true surface. The hypothesis of a yellow or red soil or rock it will be convenient to retain, to provide the yellow dust with which to impregnate the dense atmospheric shell and give it the color necessary to explain the pictures. Under these assumptions, the apparent white surface which we see is imagined to be a uniform shell of light cirrus clouds overlying a dense, yellow atmosphere. On occasions of violent atmospheric disturbance the uniform cloud-covering is broken up, and we see the underlying yellow atmosphere, to which

are due the dark markings seen visually and photographically. Or the cirri may in certain regions be piled up in heavy masses, again seen visually and photographically as brilliant white clouds.

The observed visual albedo of Venus is not out of harmony with the hypothesis of a covering of light cirrus clouds. The coefficient of reflection of dense white clouds is 78 per cent. According to Russell,[1] the coefficient of reflection of the outer cloud-mantle of Venus is 49 per cent. (This is Russell's $p$ coefficient, and must not be confused with the Bond albedo, which is 0.59.) The clouds on Venus thus have considerably less reflectivity than white clouds on the earth, a fact in harmony with the hypothesis of a scarcity of water vapor, a matter to be considered later.

With light clouds overlying a yellow atmosphere, a pure-white color would not be expected. It is usually considered that the color of Venus is white. Concerning this point William H. Pickering says:

> The clouds of Venus owe their lemon-yellow color to the same cause—a very dense atmosphere. This color is strikingly shown if we view Venus by daylight in the telescope when our terrestrial clouds are crossing its disc.[2]

In 1908 and in 1921, Dr. V. M. Slipher, at the Lowell Observatory, in two series of careful experiments, failed to detect either oxygen or water vapor in the spectrum of Venus. The latter series was made under exceptionally favorable winter conditions, giving a minimum of water vapor above Flagstaff. Dr. Slipher interprets his results as follows:

> Either the atmosphere of Venus does not contain moisture in amount comparable with that of our driest winter air, at Flagstaff, or if it does (and this seems the more probable), the sunlight Venus sends us is not affected by the moisture bearing layer of her atmosphere.[3]

Professor H. N. Russell concludes:

> It is therefore likely that there is a more or less abrupt decrease in the haziness of Venus' atmosphere at a height of about 4000 feet above her apparent surface. The transition between the lower hazy and the upper clear layers is not sharp. . . . .[4]

From the point of view of the present paper, the 4000 feet represents the thickness of a shell of dust-laden yellow atmosphere, and the

[1] *Astrophysical Journal*, **43**, 190, 1916.

[2] *Publications of the Astronomical Society of the Pacific*, **37**, 12, 1925.

[3] *Lowell Observatory Bulletin*, **3**, 88 (No. 84), 1921.

[4] *Astrophysical Journal*, **9**, 297, 1899.

"upper clear layers" the shell containing the clouds. By "apparent surface" Russell undoubtedly means the lowest limit to which light penetrates.

In an important paper on the atmosphere of Venus, St. John and Nicholson[1] give spectroscopic data which are invaluable in any study of the atmosphere of Venus. Briefly, they find:

> It is estimated that in the path traversed by the solar light through Venus' atmosphere there must be less than the equivalent of one meter of oxygen, less than one-thousandth of that in our atmosphere, and less than one millimeter of precipitable water vapor.

This result, of course, does not negative the existence of oxygen and water vapor in the atmosphere of Venus, since the length of path traversed by the solar light through Venus' atmosphere is unknown. The authors say in this connection:

> If, however, the reflecting surface consists of a permanent layer of cirro-stratus, the quantity of water vapor traversed by the reflected beam would be small, as cirro-strati are formed in the upper troposphere where the temperatures are very low. These low temperatures practically insure that cirro-strati consist of ice crystals, but as at such low temperatures there is little water to condense, their structure is more or less tenuous and fibrous. . . . .

Again:

> Under the probable extremes of temperature on the opposite hemispheres of Venus it is conceivable that the violent atmospheric circulation would cause clouds of dust to be permanent features of the planet's atmosphere, dust composed of highly reflecting material such as that to which the high reflecting power of Vesta is due.

Equally valuable data are furnished by the radiometric observers. Pettit and Nicholson find that

> two methods of estimating the temperature of emission of the radiation from the dark side agree in assigning a value near 250° C. absolute. This is consistent with what might be expected from radiation emitted by a cirrus cloud covered atmosphere. . . . . These three facts of observation indicate a nearly uniform temperature over the planet's surface, both on the illuminated and dark hemispheres.[2]

This last sentence is a terse statement of what is perhaps the most valuable single discovery ever made with respect to the planet Venus. Startling at first sight, it can be considered to be more or less a quantitative proof of the great density and extent of the at-

[1] *Mt. Wilson Contr.* No. 249; *Astrophysical Journal*, **56**, 380, 1922.

[2] *Popular Astronomy*, **32**, 614, 1924.

mosphere of Venus. For the atmosphere of Mars, known to be of limited extent, gives radiometrically a difference of temperature of the order of 100° C. between the bright and dark sides. Pettit and Nicholson consider that their observations prove the existence of an isothermal layer over Venus, the temperature of which is −23° C., similar in properties to the isothermal layer surrounding the earth.

If the period of rotation of Venus is 225 days, as many observers have been led to believe, it is difficult to see how this high temperature of the radiating layer of the night side can be maintained. A period as short as two weeks would require an atmosphere of considerable density to maintain the temperature at this point during the night. That the temperature of the unilluminated disk is relatively high has been confirmed by Coblentz and Lampland[1] from observations taken only a few weeks later than those by Pettit and Nicholson.

*Spectroscopic evidence of rotation.*—The first spectroscopic determination of the period of rotation of Venus which can be considered as of the highest order of precision is by Dr. V. M. Slipher.[2] He obtained a negative velocity of −0.005 km/sec., with a probable error of ±0.04 km/sec. Taken by itself, this means that it is an even chance that the period of rotation is greater than fifteen days. A mass of unpublished material obtained at Mount Wilson by Adams, and by St. John and Nicholson, is so completely confirmatory of Slipher's result as to leave little doubt of the final spectroscopic verdict. St. John has said in a personal letter: "If we assume that the rotation is direct, the chances are about 10 to 1 that the period is longer than 20 days. If, however, the retrograde rotation is real, the chances are about 10 to 1 that the period is longer than 6 days."

Pettit and Nicholson,[3] from their radiometric observations quoted above, find some slight evidence of a direct rotation of an undetermined amount.

The evidence of rotation furnished by the markings seen by visual observers is so utterly contradictory that no confidence can be placed in this method of attack. Some data of a physi-

[1] *Journal of the Franklin Institute*, **199**, 804, 1925.

[2] *Lowell Observatory Bulletin*, **1**, 9, 19 (Nos. 3 and 4), 1903.

[3] *Loc. cit.*

cal nature, however, suggest a comparatively short period of rotation—just how short it is unfortunately impossible to state. While lacking the precision of the spectroscopic method of obtaining the rotation, these data cannot be ignored; and they will undoubtedly furnish material for discussion and debate for many years. Briefly summarized, they are as follows:

*a*) The high temperature of the night side of the planet found by Pettit and Nicholson, and independently by Coblentz and Lampland.

*b*) The persistence of the parallel bands or markings, perpendicular to the terminator, as shown on the photographs (Plates I and II), which is suggestive of at least a moderate speed of rotation around an axis coinciding approximately with the plane of the orbit. It is difficult to see how markings of this type can be otherwise explained. With a long period of rotation, the markings should favor a symmetry around the subsolar point. Granted the strong insolation on Venus and an extensive atmosphere, the latter necessary to maintain the high night temperature, the observed belted appearance, presumably an atmospheric phenomenon, requires for its maintenance, I believe, a velocity of rotation certainly of an amount considerably in excess of that resulting from a 225-day period. Clayden says:

> There is no question that the earth is a belted planet; and if Venus had a rotation period comparable with our day, and if her atmosphere is of similar or greater density than ours (which is essential for the formation of the shell of convection clouds), she also would take a place in the same group.[1]

From the apparent lack of these belts, Clayden concluded:

> There seems, therefore, no reasonable doubt that if the polar axis of Venus forms a large angle with the plane of the orbit, her rotation period cannot be anything like so short as the length of our own day.

And further (p. 198):

> Probably any period between 20 of our days, on the one hand, and, say, 180 or 200 days, on the other, would give conditions in which no cloud bands or high-pressure tropical belts would be produced parallel to the equator, and no permanent locking up of the water could occur.

And (p. 200):

> Since atmospheric pressure should, on the whole, be considerably greater on the dark side than on the one turned towards the sun, the upper currents should flow from the hot side over the high-pressure ring and towards the terminator, while colder under currents should flow in the opposite direction.

[1] *Monthly Notices, R.A.S.*, **69**, 197, 1909.

*c*) It appears to me difficult to reconcile the rapid changes shown by the present series of photographs to be taking place in the atmosphere of Venus with a rotation period of any considerable length. If the period were, for example, 225 days, there is no doubt that a steady atmospheric state would be reached, in which all detail would be obliterated. The observed changes appear to be greater than those occurring in the earth's atmosphere, at least in the temperate zones. Even on Mars, with its thin atmospheric covering, persistent cloud formations are at times noted. Thus in 1926, at Mount Wilson, I obtained photographs in ultra-violet light of a cloud formation on Mars, which persisted for a period of at least three days.[1] It is to be noted that this was of the same type as the clouds shown on the present series of photographs. Visual observers, notably W. H. Pickering, have, however, observed markings which apparently persisted for long periods of time, from which they have deduced values of the rotation period.

In the absence of essential data, the reasons for a short period, excepting the motions of markings, are thus based largely on intuitions. A compromise must be made with the spectrographic data, which strongly negative a period as short as fifteen days, but conflict less strongly as the period admitted is lengthened. Since the spectroscopic objection should be quite feeble if the period is set at thirty days, I believe that this is the best compromise possible at the present time.

*Rotation period determined from markings.*—Markings on Venus were first detected at Naples, in 1643, by Fontana, who was also the first to see the surface features of Mars (1636). From the apparent motion of these markings he deduced a rotation period of twenty-three hours and twenty-one minutes. This short period was later confirmed by Schröter and De Vico. From a long series of observations of the markings, both by himself and others, Schiaparelli was convinced that the rotation period coincided with the planet's year, and that the axis was not far from the pole of its orbit. Since Schiaparelli, a host of observers, mostly with small telescopes, have noted markings, both light and dark, on the surface of Venus. The general character of the dark markings as drawn by these ob-

[1] *Astrophysical Journal*, **64**, 245, 1926.

servers has been almost uniformly the same. The markings are long and narrow, favoring a direction parallel to the terminator, with the exception of shorter markings which are sometimes drawn running in from the terminator, nearly perpendicular to it. Notable exceptions are the drawings of Lowell, who observed a series of long narrow markings radiating from the subsolar point, which he believed to be features of the surface of the planet, formed by cyclonic inrushes of air from the cold night side to the intensely hot subsolar region, along certain favored paths governed by the original topography of the planet. Micrometric measurements of the junction point of the markings, extending over a period of 40 days, were consistent with a rotation period of 225 days, the orbital period.[1] Expressing doubt in 1902 as to the reality of these markings, Lowell[2] later returned to his original opinion as to their reality and significance, which is outlined above.[3]

It is natural to assume that the markings observed visually are closely related, in position and general form, at least, to those shown in Plates I and II. This view is strengthened by the fact that both Quénisset and Rordame were able to see visually the markings which they photographed. It seems to me to be of great significance, however, that of the many hundreds of markings which have been seen and drawn by visual observers, so few are cases of the banded structure, perpendicular to the terminator, a characteristic feature of the present series of photographs. W. H. Steavenson[4] observed a pair of this type on March 9, 1924. E. C. Slipher in a personal letter writes: "A hazy band-like shading often showed to me near the middle of the disc (equatorially) and many times this appeared roughly and irregularly double." While this general lack of agreement apparently casts doubt on the relationship between the visual and photographic markings, I am averse, on physical grounds, to imagining that they are entirely distinct. The upheavals which are conceived to take place in the lower atmosphere must produce simultaneous phenomena in the hypothecated upper

[1] *Monthly Notices, R.A.S.* **57**, 402, 1897.

[2] *Astronomische Nachrichten*, **160**, 130, 1903.

[3] *Popular Science Monthly*, **75**, 521, 1909.

[4] *Journal of the British Astronomical Association*, **36**, 298, 1926.

yellow atmosphere and in the overlying mantle of cirrus clouds. That these upheavals are of enormous extent is proved by the photographs.

It is unfortunate that a series of visual observations could not have been obtained simultaneously with the photographs. On each evening on which photographs were taken, before the exposures were started, a careful search was made visually for markings. On the evening of June 30, with seeing excellent, the surface was carefully scanned, by both Professor Russell and myself, without anything being noted. Blue, green, yellow, and red filters were tried. It is quite probable that the lack of success was due to too great contrast between Venus and the sky background, which produced the phenomena accompanying glare, in which the discriminating power of the eye for differences in shade is very much reduced, the amount depending upon the degree of the glare.

It is to be noted that glare cannot be defined solely with reference to the relative intensity of object and background. Its exact definition is exceedingly difficult. The amount of dark adaptation of the eye is of paramount importance. The claims of those observers who are able to see markings on Venus, namely, that it is necessary to make observations in full daylight, are therefore not without support. Even in the evening or morning twilight, the visibility of the markings may very well be less than in full daylight. On account of the relatively low intensity of the light from the sky at Mount Wilson and at other observatories situated at great elevations, the glare of Venus, even in daylight, should be much greater than at sea-level, so that the visibility of the markings may very well be less. There is the further factor of magnification, discussed on page 65. It is noteworthy that at the Lick Observatory Professor E. E. Barnard was able to see markings on Venus distinctly on only one occasion. He says:

> Venus was frequently observed with the 12-inch refractor at Mt. Hamilton during the years 1888–95, but I never could (with but one exception) satisfactorily see the markings. Vague, indefinite spots were often visible, but it was not possible to see them well enough to identify them for rotational purposes. . . . .
>
> The 29th of May, 1889, was remarkable for the thickness of the atmosphere from smoke and dust. One could scarcely see across the canyon so dense was

the haze. Examining Venus with the 12-inch on this date I was struck with the remarkably perfect definition. There was not the slightest tremor of the image.[1]

While Barnard attributed the visibility to the perfection of definition, I would attach considerable importance to the sky brightness augmented by dust and haze.[2] He says further: "The planet was watched through many years, but indifferent seeing always baffled one and no satisfaction could be gotten out of it."

A drawing of the markings seen on May 29 is given by Barnard. Apparently they consist of three segments of a long streak parallel to the terminator, but, on account of the narrowness of the crescent, they might very well be three foreshortened bands parallel to the equator. Later, from the Yerkes Observatory, Barnard writes as follows:

> At times in the early summer here the seeing in the day is very perfect. Venus has been examined on a number of such occasions when the planet was beautifully defined. Surface markings were nearly always present, but they were very illusive and at no time could a satisfactory drawing be secured. The visible markings were large dusky spots essentially like those . . . . on Mercury but much fainter.[3]

In view of the experience of Professor Barnard at the Lick Observatory in observing Venus visually, it is not surprising that I was unable to see markings. It was not possible to attempt daylight observations, since the consequent heating of the 60-inch mirror would have put it in poor condition for the night observing. It is hoped that the many European observers who make a practice of observing Venus may have obtained satisfactory drawings of the markings during the interval covered by these photographs, especially on the dates of the strong bandlike markings of the present series. Comparison with the photographs should prove instructive as well as valuable.

[1] *Astrophysical Journal*, **5**, 300, 1897.

[2] It is well known that fine seeing generally accompanies a hazy or smoky atmosphere. While this does not mean that they are necessarily cause and effect, I would suggest that they may be. Rapid vertical movement of the air, resulting in an excessive adiabatic cooling, is fatal to good seeing (see Dr. J. Halm, *Proceedings of the Royal Society of Edinburgh*, **25**, 458, 1906). Dust and smoke can be conceived as mechanically loading the air, cutting down the velocity of convection. In addition, by absorbing and emitting radiation, they would tend to equalize temperatures and restore the thermal equilibrium upon which good seeing depends.—NOTE BY F. E. R.

[3] *Astronomische Nachrichten*, **157**, 263, 1902.

The period of rotation and the position of the axis of Venus found by Professor William H. Pickering[1] are worthy of special notice. His results are based upon the apparent motion of two very conspicuous spots, which were followed for a period of thirty-four days. He finds a period of about sixty-eight hours, and that the axis lies approximately in the orbital plane, in heliocentric longitude of 47°, the direction of rotation corresponding to a rolling motion. Pickering's values, both for position of axis and period of rotation, were confirmed by H. McEwen.[2] W. H. Steavenson confirms Pickering's axis, but finds the period to be eight days.[3] The rapid rotation found by Pickering and McEwen is not inconsistent with Slipher's negative spectrographic result, for, as pointed out by Worthington,[4] Slipher's observations were made at a time when the axis as above located passed very close to the earth. On the other hand, the unpublished spectrographic observations by St. John and Nicholson, taken at a time when the axis as located by Pickering did not pass near the earth, fail to show rotation. During the latter part of June, 1927, when the present series of photographs was made, the axis, as thus determined, passed close to the sun, and the earth lay near the plane of its equator, so that the motion of spots should have been from north to south. If the period is sixty-eight hours, the wedge-shaped marking so conspicuous on my photographs of June 26 should have moved downward 1.2 mm during the interval of forty-eight minutes between exposures *a* and *g*. No such motion is apparent, although it is not impossible that it may have occurred, since 1 mm is approximately the limiting motion on the scale of the prints which this marking is capable of showing. If it be assumed that motions of markings of this nature may be relied upon to give the rotation period of the planet, all that can be concluded from the present series of photographs is that the definitely bounded marking of June 26 indicates a period of rotation which must be greater than about three days. But, on account of the great diversity of rotation periods and axes which have been derived from the motions of visual markings, it is quite evident that no reliance can be placed on results thus obtained, unless there are abundant confirmatory

[1] *Journal of the British Astronomical Association*, **31**, 218, 292, 1921.

[2] *Ibid.*, **36**, 191, 1926. [3] *Ibid.*, **36**, 297, 1926. [4] *Ibid.*, **31**, 221, 1921.

data. We have only to glance over the present series of photographs, noting the great changes in cloud formation from day to day, to be convinced of the futility of the method when applied to individual markings. I have no intention of discouraging the observation of markings, either visually or photographically. The motions of clouds and their changes in form, if observed systematically, will doubtless lead eventually to valuable data on the problems which confront the student of Venus.

*Clouds at the north and south points.*—The bright regions at the north and south points, especially strong at the latter, were first seen by Schröter (1789–1790), who ascribed them to enormous mountains (42 miles high), protruding beyond the atmosphere. They were extensively observed by Trouvelot (1877–1882), whose pen picture is worthy of preservation:

> The polar spots are distinctly visible, the southern one being the more brilliant. Their surface is irregular and seems like a confused mass of luminous points, separated by comparatively sombre intervening spaces. This surface is undoubtedly very broken, and resembles that of a mountainous district studded with numerous peaks, or our polar regions, with numerous ice-needles brilliantly reflecting the sunshine.

Trouvelot, like Schröter, believed that high mountain masses were involved. In this connection, Ellen M. Clerke, in a booklet entitled *The Planet Venus*, says (p. 36):

> Photographs of the transit of Venus, taken in Mexico by MM. Arago and Bouquet de la Grye, on Dec. 6, 1882, show a great bulge on her side toward the southern pole, establishing the fact, in the words of the latter observer, that "there exists on the south of the planet a zone characterized by a great elevation between two depressions."

Commenting on the older explanation of immensely high mountains she says (p. 37):

> There is however another possibility, and they may not be solid rock structures but cloud masses piled up to an enormous height, perhaps at the meeting point of warm and cold air currents.

There appears to be no doubt of the existence of these bright areas, which on occasion are seen to cover one or both presumed poles. They have been reported by numerous observers, are confirmed by the present series of photographs, and are therefore undoubted facts to be reckoned with. It is an attractive hypothesis

to assume that they mark the atmospheric pole, being similar in nature to the atmospheric polar cap of Mars, which has been photographed and discussed by Wright.[1] Further careful study of this feature of the planet's surface will doubtless be fruitful in exposing conditions existing in the upper atmosphere and in furnishing indirect evidence of the rotation period and location of the poles. The persistent appearance of bright clouds at the north and south points and nowhere else along the terminator undoubtedly marks or labels them as preferential points, with the probabilities in favor of their being the polar points. Their existence, equally with that of the equatorial bands, is suggestive of a comparatively short rotation period. No doubt spectrographic observers, in the search for water vapor in the atmosphere of Venus, will, at some time or other, be so fortunate as to secure spectrograms when these clouds are unusually strong. The slit of the spectrograph should be set tangent to the disk at the cusp, in order to obtain the maximum absorption.

It is exceedingly suggestive that Coblentz and Lampland[2] obtained a higher temperature for the southern half of the disk than for the northern. There is evidently a correlation between temperature and bright clouds, the exact nature of which is an attractive problem for the future.

The south polar cloud is unusually strong on the photographs of June 6. It is also strong on those of several days later in the month. During July it is not generally visible, the northern cusp now showing stronger. It is evident that the phenomenon is more or less ephemeral, the persistence of the atmospheric cap of Mars being lacking. It is quite likely that the clouds disappear for long periods of time, just how long it would be difficult to say. It is significant that they were apparently never seen by Barnard or Lowell. It appears, however, from a letter by Dr. V. M. Slipher to me that Lowell actually did observe them at times. E. C. Slipher, concerning his own observations, writes me: "Sometimes polar brightening existed, while at other times, none were seen. Generally speaking, these have been most often observed at the south."

In addition to the polar cloud, there is one other feature of the

[1] *Lick Observatory Bulletin*, **12**, 59 (No. 366), 1925.

[2] *Journal of the Franklin Institute*, **199**, 805, 1925.

drawings and photographs of Venus upon which many observers agree, namely, a dark marking perpendicular to the terminator, and adjacent to the bright south polar area. It is well seen on the photograph of June 6*a*. It appears on Quénisset's drawing made from his photograph of February 23, 1921,[1] as well as on the drawings of Schiaparelli, Trouvelot, and Lowell. Lowell has described it at length, referring to it as a collar, with spots or enlargements strung upon it, like beads. Lowell does not appear, however, to have observed the polar cloud, to which the collar is the border. Whether this is really a favored region for the development of one of the system of bands, as seems likely, must be left to future observation.

*The terminator*—A third phenomenon worthy of serious consideration in a study of the atmosphere and rotation period of Venus is to be found in the appearance of the terminator, which is unusual and exceedingly suggestive. Ellen M. Clerke[2] speaks of the dark band adjoining the terminator as "a gray segment of shadow," explaining it in the following words:

> The higher clouds being . . . . always nearest the sun, would cast their shadows on the lower banks to a sufficient extent to dull the luminosity of the inner edge, on which the solar rays are very oblique.

The breadth of this band is much greater than would follow from any reasonable law of scattering and reflection in a homogeneous atmosphere. According to Clayden, it is more pronounced at eastern than at western elongation. I can do no better than to quote him:

> . . . . The acceleration of phase in eastern elongation is commonly observed to vary from 4 to 8 days,—dichotomy, or any special phase being so much earlier than the theoretical time. This can only mean that the surface along the sunrise terminator is not at right angles to the planet's radius, but that it slopes upward toward the sun. . . . . But this is exactly how the upper surface of a shell of rising convection clouds should rise, and the fact alone is an almost conclusive proof that the cloud theory is right. . . . . The retardation of phase in western elongation is rather more pronounced. It can be explained as due to an even greater slope of the mean surface of the clouds, due to the collapse of the rising currents and breaking down of the clouds towards sunset. On the earth the fall of the cloud levels in the evening is distinctly more rapid than their morning rise.[3]

To the writer, visually, the color of the broad band of shade adjoining the terminator was distinctly brown. As far as I can find,

[1] *Comptes Rendus*, **172**, 1645, 1921.
[2] *Op. cit.*, p. 31. [3] *Op. cit.*, p. 202.

only one other observer speaks of the color. W. Reid says: "I have also noticed that there is generally a brownish border to the terminator."[1] The failure to see the color at observatories nearer sea-level may be due to the white atmospheric glare, observations generally being made in the daytime. This would not apply to the observations of Barnard and Lowell. E. C. Slipher states in a letter that he has not noted the color. Whether it is a permanent feature or not is therefore a matter for future observations to decide. The brown color fits in very nicely with the assumed yellow atmosphere underlying the shell of cirrus clouds. The supporting remark of Reid removes the possibility of personal bias. The dark markings, observed visually, should be yellowish. On account of their extreme faintness, it is, however, too much to expect that any color would be noted, since color perception diminishes rapidly with intensity and angular size. The reader is reminded that the present series of photographs proves that the dark markings are yellowish, since their visibility in the photographs has been shown to increase toward the short waves.

It is significant in this connection that the dark bands which have been photographed always start from the terminator. This is not a characteristic of the dark markings drawn by visual observers, as has been noted above. Clayden[2] has called attention to the fact that the band parallel to and adjoining the terminator must be a high-pressure ring, and therefore deficient in cirrus clouds. From the same viewpoint, since the dark markings or bands are physically connected with the terminator, they are to be considered as regions of high pressure, and accordingly free of cirri. The contiguous areas are the regions of low pressure in which cirrus and cumulus clouds prevail, the alternations being very similar to those existing on the earth.

*Previous photographs of the markings on Venus.*—As far as I know, the first to obtain photographic evidence of markings on Venus was F. Quénisset,[3] using an astrographic refractor, in conjunction with an enlarging lens. Quénisset states that his first successful photographs showing markings were taken June 29, 1911. On

[1] *Journal of the Astronomical Society of South Africa*, **2**, 43, 1927.

[2] *Op. cit.*, p. 203. [3] *Comptes Rendus*, **172**, 1645, 1921.

February 23, 1921, he obtained photographs of two large dark areas bordering the terminator, one equatorial and one near the south cusp. On the following night they had disappeared, smaller markings taking their place. The photographs, however, were not shown, no doubt on account of the difficulty of reproduction due to the extreme faintness of the markings.

I believe that the first reproduction of photographs of markings on Venus was by Alfred Rordame.[1] These were taken at Salt Lake City on July 17, 1921, in full daylight, on commercial film, behind a yellow filter. A 9-inch visual refractor by Clarke, with enlarging camera, was employed. Mr. Rordame kindly sent me his original negative for examination, which shows very much more detail than the reproduction would lead one to expect. The width of the dark equatorial band shown on his negative is about one-quarter of the diameter of the disk, and is full of curious detail. It makes an angle of about 60° with the terminator, bearing north, stopping considerably short of the limb. A bright cloud borders it on the south near the terminator. A parallel and similar marking, but much smaller, is seen near the northern cusp. The cusps are free from clouds. Rordame was able to see these markings visually. The effective wave-length of the photograph is, however, uncertain, since the ordinary kodak film used by Rordame is to a certain extent orthochromatic. This uncertainty, combined with the great variation in "cut" of the commercial yellow filter, makes the wave-length indeterminate. It is quite likely, however, that these markings are of the same class, and, accordingly, have the same effective color as the markings shown in the present paper. These, as previously noted, are strong in the ultra-violet, and diminish in intensity to such an extent with increasing wave-length that only a very faint impression is obtained at an effective wave-length in the neighborhood of 4700 A (p. 64). Since the effective wave-length of Rordame's negative is probably around 5000 A, the conclusion is either that he was so fortunate as to observe a marking of very exceptional strength or that the effective wave-length, or the color, of the marking which he photographed lies in the yellow region of the spectrum. This would correspond to the rare yellow clouds over Mars.

[1] *Popular Astronomy*, **30**, 137, 1922.

On account, however, of the similarity in appearance to the violet markings shown in Plates I and II, I am disposed to accept the first alternative. These violet markings have been interpreted physically as variations in structure of the thin layer of cirrus clouds which overlie the dense yellow lower atmosphere, due undoubtedly to violent disturbances originating far below, perhaps near the surface of the planet itself. There is no doubt that there should be some evidence of these storms within the underlying yellow atmosphere, either shadow effects or effects due to a mixing of layers of atmosphere of different concentrations or composition. We should then have the two classes of clouds found by Wright to exist on Mars: (*a*) the common violet cloud, and (*b*) the rare yellow cloud. The relation between the two should be more intimate in the case of Venus, since they are assumed to be produced by the same underlying disturbance.

Professor William H. Wright, in taking photographs of Venus in blue-violet and in infra-red light, obtained markings on the former. Since the published reproduction[1] does scant justice to the negative, Dr. Wright, at my request, kindly sent me his original negatives, taken October 10, 1924, for examination. They show features not appearing on any of my negatives, namely, two long *bright* bands, one on each side of the presumed equator, the southerly one being the stronger of the two. Wright describes the markings as ". . . . two nebulous brightish streaks, or bands, running in a direction approximately at right angles to the terminator."[2] Unlike the dark bands, they do not start at the terminator. As noted by Wright, the inclination to the equator is not great, and here again the banded character of the markings is seen. On Wright's negatives, the cusps are weak. On none of my photographs do the *bright* clouds show as bands, as on Wright's. They are, on the contrary, more or less circular or irregular areas, which favor the regions of the cusps. Professor Wright has, in addition to the foregoing, sent me a print from a negative of Venus taken by him on November 23, 1927, which shows a banded cloud formation quite similar to that on my negative of June 24. The regularity

[1] *Publications of the Astronomical Society of the Pacific*, **39**, 220, 1927.

[2] *Loc. cit.*

with which the different configurations repeat themselves is a matter of the highest importance for future study.

### DIAMETER OF VENUS

Probably no branch of observational astronomy is so full of pitfalls as is the measurement of the diameters of celestial objects, in particular, those of the sun, moon, and planets. Dr. W. W. Campbell[1] gives a fine summary of the many sources of error entering into visual measurement of diameters. In general, diameters measured visually with a filar micrometer are too large, due to irradiation. Measured with double-image instruments, such as the heliometer, they are too small. It is customary to correct this error, assumed as constant, by making the measurements over the greatest possible range of distance of the object, all conditions being kept constant.

In 1926 I made a series of laboratory experiments to test the accuracy of photographic measurements of diameters.[2] The results indicated that great reliance can be placed on measurements of this kind, made under widely varied conditions, such as method of illumination of the object, kind of plate, and development. With under-exposure the image is too small, while an over-exposed image gives too large a diameter. This would be disconcerting were it not for the fact that there is a certain range of exposure within which little or no change in diameter is found.[3] This range may be considered as the region of correct exposure. Unfortunately, the laboratory results do not apply to astronomical exposures. Just as star images increase in diameter progressively, starting from threshold exposure, so also do the images of the disks of the planets increase progressively with increasing exposure. There is, therefore, no value of the exposure which can be recognized as giving the true diameter, as was the case in laboratory measurements.

The systematic errors in measurements of the photographic diameter of Mars have been studied by William H. Wright,[4] by R. J. Trumpler,[5] and by P. Van de Kamp.[6] Trumpler finds that the dependence of diameter on the density of the image is small, and

[1] *Astronomical Journal*, **15**, 145, 1895.

[2] *Astrophysical Journal*, **64**, 247, 1926.

[3] *Ibid.*, **52**, 100, 1920.

[4] *Lick Observatory Bulletin*, **12**, 56 (No. 366), 1925.

[5] *Ibid.*, **13**, 27 (No. 387), 1927.

[6] *Astronomical Journal*, **38**, 61, 1928.

that the constant correction, for the density chosen as a standard, is negative, that is, the measures are too large. Van de Kamp, on the other hand, finds a very great dependence of diameter on density of image, and that, for the density chosen as standard, the constant correction is large and positive, the measured diameter being too small. It seems quite likely that the coarse-grained plate (Cramer Isochromatic) used by Van de Kamp is the chief cause of his large density coefficient. He says (p. 69):

> The effect of the incomplete representation of the geometrical image of the planet's disc probably is mainly due to the photographic action of developer and plate. . . . . The increase of the measured diameter with increase of photographic density on a plate must be ascribed to an actual increase of the image due to photographic spreading action.

My laboratory experiments noted above prove that the spreading of an image is not a photographic phenomenon, but an optical one. For example, in experiments made by me in the laboratory of the Eastman Kodak Company, a coarse line, 1 mm broad, formed by two opposing knife blades, with edges carefully trued and lying in the same plane, was printed by contact on various kinds of photographic plates. It was found that no spreading whatever of the print or image took place, over a very great range of exposure time.

The spreading phenomenon generally observed in astronomical photography is due primarily to lack of sharpness in the optical image formed on the photographic plate, and is aided and abetted by plate and developer. If the image is perfectly sharp, the influence of the photographic factors is small. Aside from the object itself, optical sharpness is largely a matter of telescope and seeing.

In measuring photographically the diameters of the planets, I believe that the principal source of error is to be found in the falling off in intensity of the light coming from the extreme edge of the planet. If the exposure on the planet and the subsequent development of the plate are so adjusted as to show the surface features to the best advantage, a result obtained by a rather light exposure and soft development, the limb will be under-exposed, and a small diameter accordingly obtained. It is not sufficient to calibrate by measuring the average density of the image itself, since it is the density at the edge of the disk which is of importance. The ratio, $R$, of the density at the edge to the average density over the disk

is constant only for standardized development and seeing. If it be granted that all these conditions are satisfied, and the important constant error determined by measuring the diameters when the planet is near and far off, respectively, the result is still open to question, since it is quite likely that the ratio of the intensity of the light coming from the edge of the planet to that from the disk, or $R$, depends upon the angle subtended at the sun by the planet and earth, respectively. In the case of photographs taken in infra-red light, $R$ appears to be very small, so that the determination of the diameter in infra-red light becomes very troublesome. Professor Wright was the first to show its importance in measurements of diameters, remarking:

> In measuring the infra-red plates a different procedure had to be adopted for the reason that there is a true fading of the image as the edge is approached. If the center of the disk is normally exposed one can not be sure that the true edge is actually visible.[1]

The difficulty in the measurement of diameters due to edge fading can perhaps be overcome in the following way: Let a series of exposures be made, with the logarithms of the times increasing linearly, the steps being short, so that we obtain images with densities ranging from under-exposure to over-exposure. Let the diameters be plotted against the logarithms of the exposure times. The resulting plot should, I believe, consist of two straight lines, the intersection of which will give the correct exposure time, and, in addition, the diameter of image which is least affected by edge fading, and by irradiation. The method is based on the conception that the rate of growth of an image must change when a diameter equal to the true or geometrical value is reached. The rate of growth of the image of the planet before the true edge is reached depends upon the edge fading, or the rate of change of the intensity of the light as the edge is approached, and, in addition, upon the kind of plate and development. It is directly proportional to the linear diameter of the image, and, accordingly, varies with the geocentric distance. There will also be a phase effect, complicating the problem. The rate of growth, after the edge is passed, should, in general, be considerably less, depending upon seeing, telescope, plate, and

[1] *Lick Observatory Bulletin*, **12**, 52 (No. 366), 1925.

development. It seems quite probable that with this method, the constant error, which is determinable by variation of distance, should be considerably less, and its troublesome variation with color and phase much less marked. Fine-grained plates should be used, in conjunction with "hard" development. It is important that the seeing should be good, and the focusing very carefully done.

On account of the phenomenon of edge fading and its variation with phase and geocentric distance, I doubt very much the applicability of the usual method of determining a "constant" error by variation of distance. Considerable research on the subject is needed.

In measuring diameters photographically, there must be a strong color effect. The great difference between the intensity of the light coming from the edge of a planet and the average intensity over the disk is well brought out by Wright in interpreting his very fine photographs of Jupiter, made in light of different colors.[1] It is accordingly necessary to determine the constant correction to the diameter separately for each color. The photographic determination of diameters thus becomes a very troublesome matter. If, however, the dependence of diameter on the density of the image can be kept small, and Trumpler (*loc. cit.*) has shown that it can be, results of value should be obtained. It seems to me that the effect of phase noted above is the largest outstanding source of uncertainty, since it upsets the constancy of the conditions necessary in determining the constant error for each color.

The diameters of Venus derived from the present series of photographs are open to many of the objections which have been enumerated. No attempt was made to determine the dependence upon density. Only those images were measured which were considered to be properly exposed, that is, to have the density most advantageous for bringing out the details on the surface or disk of the planet. In measuring the diameters, the thread was set tangent to the extreme outer edge of the penumbra of the image, since I consider this to be the image of the extreme border of the planet, the light from which is very feeble. In order to be able to see the outmost ring of the penumbra, a microscope of very low power was used in the

[1] *Publications of the Astronomical Society of the Pacific*, **39**, 358, 1927.

measurement, and the illumination of the plate was very much reduced—both factors of importance.

The usual method of measurement was found to be unsatisfactory, especially for the crescent phase of the planet. Except when clouds are present at the north and south points, the cusps are exceedingly weak photographically in all colors, especially in the red and the infra-red. In order to overcome this difficulty, the following device was used: A series of holes, having diameters corresponding to the range in diameter of the images to be measured, was bored in a brass plate. The holes were made accurately round. The image to be measured was placed film side down on the plate, a hole being selected which was a trifle larger than the image. By eye estimate, the image was placed centrally over the hole. The uniformity in the breadth of the clear ring separating the edges of the hole and image was the criterion of proper adjustment. The negative and plate were then clamped together. The clear space, or ring between the image and the edge of the hole, was measured at two points chosen as near the cusps as practicable, depending on their density. The sum of these two distances, subtracted from the known diameter of the hole, gives the required diameter of the image. The accidental error introduced by not setting the plate accurately over the hole, in the mean of a large number of plates, should be small. In any case, it enters with a reduced factor, being zero when the points chosen for measurement are diametrically opposite. It was a great advantage not to have to make settings on a pair of weak cusps, which would have resulted in diameters too small, especially in the infra-red.

On account of the many sources of error enumerated above, I have not measured the plates or made the reductions in a definitive way. The present series of photographs was exposed and developed primarily for surface detail, and not for measurements of diameter, and accordingly suffers from the technical faults which have been analyzed above. It is not considered worth while to give the detailed results. The values for the red and the infra-red are averaged, since they were in excellent agreement, and also the values with and without blue filter. Diameters of images obtained with the 100-

inch mirror are not included, since the scale value for it was not determined directly. The values given are for unit distance.

| | No. Nights | Diameter | Probable Error |
|---|---|---|---|
| Red and infra-red | 12 | 16″.82 | ±0″.030 |
| Blue-violet | 13 | 16.88 | .046 |
| Ultra-violet | 13 | 17.17 | ±0.043 |

The smaller probable error for the red diameter results from the greater sharpness of the images, due, in a measure, to the fact that they were made when the zenith distance of Venus was less. It is to be noted that the infra-red diameter is 2 per cent less than the ultra-violet diameter. This is smaller than the corresponding difference which I obtained in 1926 for Mars,[1] which was 6 per cent. The larger difference in the case of Mars was undoubtedly due to the coarse-grained plate used for the ultra-violet pictures, necessitated by the slow-working filter employed in that series. The difference found above between the diameters in red and in ultra-violet light

TABLE II

DIAMETERS OF VENUS

| | Diameter | Constant Error |
|---|---|---|
| E. Hartwig, Breslau heliometer | 17″.67 | −1″.23 |
| Oxford observations, reduced by Hartwig | 17.58 | 1.18 |
| Kaiser's observations, reduced by Hartwig | 17.41 | 0.77 |
| L. Ambronn | 17.71 | 0.79 |
| J. I. Plummer, Durham | 17.32 | −0.55 |
| A. Auwers, transits of 1874 and 1882 | 16.80 | ..... |
| E. E. Barnard (night observation, at Lick Observatory) | 17.40 | ..... |
| E. E. Barnard (daylight, at Yerkes Observatory) | 17.14 | ..... |
| W. H. Pickering (daylight at Arequipa) | 17.03 | ..... |

is due to two causes: (*a*) differences in penetration into the atmosphere of Venus of the short and long waves of light which produce the ultra-violet and red pictures, respectively; (*b*) edge-fading, which affects the diameters systematically, as explained on page 86, and which is greater in the case of the red pictures. It is, unfortunately, impossible to make any estimate of the relative importance of these two factors. The method outlined on page 85 should, however, eliminate the effect of edge-fading, thus allowing some estimate to

[1] *Astrophysical Journal*, **64**, 248, 1926.

be formed of the actual differences in diameter for light of various wave-lengths.

It is of interest to compare the diameters of Venus obtained above, with previous determinations, which were all made by visual methods. They have been compiled from various sources, and are collected in Table II.

All visual methods of observation are represented in this table. The sign of the constant error given for the first five determinations indicates that diameters measured with the heliometer or with double-image instruments are always too small. Barnard's and Pickering's measures were made with filar micrometers, and are presumably too large, on account of irradiation. Dr. W. W. Campbell[1] found for his daylight measures of the polar diameter of Mars a negative correction of 0″.05. Since Venus is considerably brighter than Mars, it seems likely that Barnard's and Pickering's values are affected slightly by irradiation.

It is noteworthy that all five values in Table II which were determined by heliometer or double-image instruments are large, the mean being 17″.54;[2] and, furthermore, that the mean constant correction, 0.90, is eighteen times the value found by Campbell for Mars, and certainly many times that which should be applied to the observations of Barnard and Pickering. I have given reasons on page 86 for believing that the application of a constant correction is of doubtful validity. Its effect is more serious the larger the correction applied. It is likely, in view of the remaining data which have been presented, that the true diameter is not far from 17″.00. With solar parallax 8″.806, this leads to a linear diameter of 7651 miles (12,313 km). With the reciprocal of the mass of Venus taken as 403,500,[3] the mean density becomes 0.919, earth being unity. If 10 miles be adopted as an estimate of the depth of the impenetrable layer of Venus' atmosphere, the density becomes 0.926. These values

[1] *Astronomical Journal*, **15**, 148, 1895.

[2] Taking account of the "contrast-theory" of A. Kühl, in which the systematic error of measurement is not regarded as a constant but is a function of several parameters, Professor Georg Struve, with the 27-inch refractor of the Berlin-Babelsberg Observatory, finds for the diameter of Venus at unit distance the value 17″.52. (*Astronomische Nachrichten*, **223**, 320, 1924).

[3] *Astronomical Journal*, **29**, 158, 1916.

of the density are notably larger than those quoted by Young, 0.86,[1] and by Russell, Dugan, and Stewart, 0.88.[2]

PHOTOMETRY OF THE DISK

Since the present series of photographs was taken largely near elongation, it was not possible to derive from them the relative albedoes of any great extent of the disk, in particular, the falling off in intensity radially from the subsolar point. For the complete study which is desirable, exposures should be made continuously from inferior to superior conjunction. Visually, the falling off in intensity or brightness of the disk of Venus, as the terminator is ap-

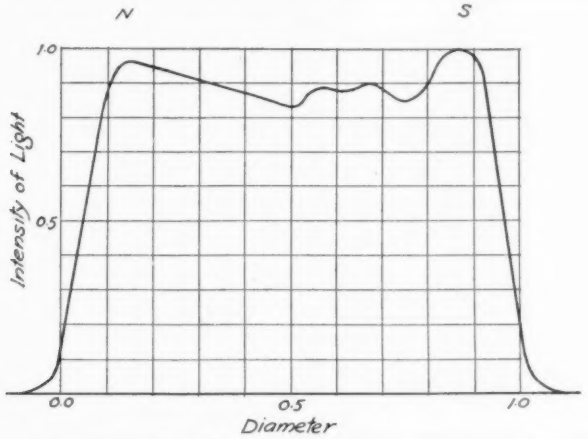



FIG. 3.—Disk of Venus: intensity-curve, parallel to terminator; ultra-violet, June 6.

proached, is very marked. Physically interpreted, this may be due to: (*a*) the relative thinness of the hypothecated layer of cirrus clouds which compose the effective reflecting surface; (*b*) shadow effects, on the assumption that the cirri are uniformly billowy. The observed high intensity of the subsolar point lends support to the second explanation. Sir W. H. M. Christie[3] has measured this brightness near inferior conjunction with a polarizing eyepiece. He found that the subsolar region was seven times brighter than the region near the limb. This seems difficult to explain on any law of reflectivity from a geometrically uniform surface, but is quite plausible, granted an upper surface formed of billowy clouds.

[1] *General Astronomy.*

[2] *Astronomy*, **1**, 315, 1926.

[3] *Monthly Notices, R.A.S.*, **37**, 90, 1876.

I am indebted to Dr. E. Pettit and Miss Louise Ware for density curves, made with the thermoelectric registering microphotometer, of a number of the negatives of Venus, a few of which were

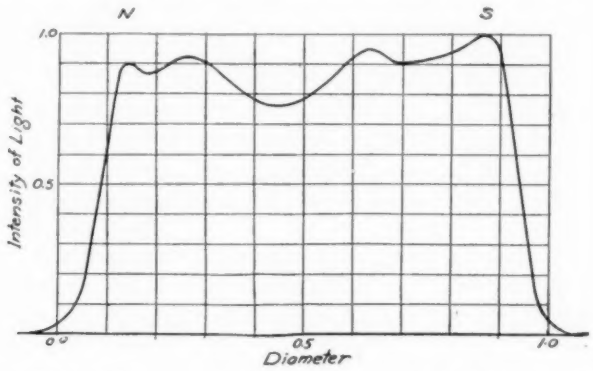



FIG. 4.—Disk of Venus: intensity-curve, parallel to terminator; ultra-violet, June 26.

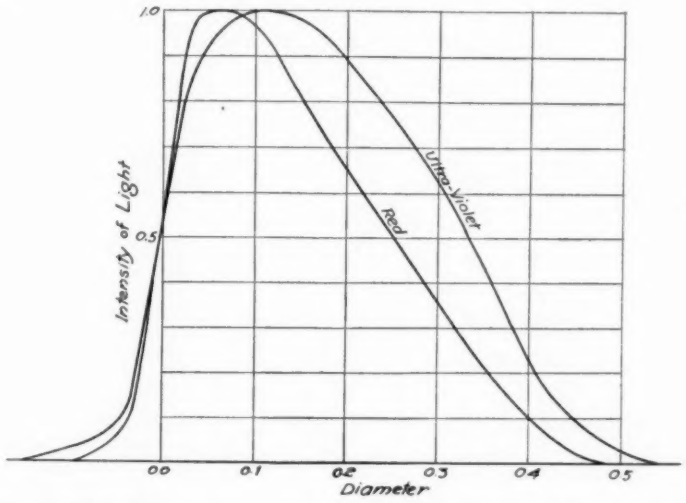



FIG. 5.—Disk of Venus: intensity-curve, perpendicular to terminator; negative of June 23.

selected for reproduction. The original curves were transformed into curves of relative light-intensity by means of a series of standard squares which had been impressed on the negatives before development (p. 61). The small fluctuations in the original curves have

been smoothed out. The width of the slit was 0.025 mm, the length 0.24 mm.

Figures 3 and 4 show the curves of intensity for ultra-violet light made from negatives Nos. 2 and 22 (Table I), which were exposed on June 6 and June 26, respectively. The section which was run is parallel to the terminator, at a distance from it of about one-third the radius of the disk. In Figure 3, the greater intensity of the southern cusp is well shown (see Plate I). The deepest shadow, corresponding to the center of a dark marking, has about 84 per cent of the brightness of this cusp. The difference between the cusps is 4 per cent. In Figure 4, the variation of intensity over the disk is more marked, reaching 24 per cent as a maximum. It is of interest to compare these values with those for negatives taken with light of longer wave-length. As already remarked, the red and infra-red negatives show no detail whatever, while the blue and the blue-violet show very weak detail. A negative made in blue-violet on June 23 was run through the microphotometer in a manner similar to the foregoing. The resulting intensity curve shows that the maximum shadows were only 4 per cent weaker than the high lights. Thus the contrast in blue-violet light is roughly only about one-fifth that in ultra-violet. This indicates in a quantitative way the importance of using the short waves in depicting detail.

Figure 5 gives similar intensity curves for ultra-violet and red light, respectively, along a section perpendicular to the terminator, obtained from negatives of June 23. The curves show that the falling off in intensity as the terminator is approached is more marked in red than in ultra-violet light. Infra-red gives practically the same curve as red. The reason for the greater decrease in intensity for the long waves is perhaps as follows: A billowy configuration for the upper cloud surface has been assumed, which will give pronounced shadows. It is these which probably cause the rapid falling off in intensity in red light as the terminator is approached. Since ultra-violet light is scattered more than red, the shadows will be less pronounced in the ultra-violet negatives than in those taken with light of longer wave.

MOUNT WILSON OBSERVATORY
YERKES OBSERVATORY
May 1928

# ON THE PRESENCE OF THE RARE-EARTH ELEMENTS IN THE SUN[1]

By CHARLES E. ST. JOHN and CHARLOTTE E. MOORE

ABSTRACT

*Astrophysical importance of the rare earths.*—These elements are at low level and ionized in the sun's atmosphere. For ionized atoms their spectra are unique in that the lines are often stronger in the arc than in the spark, and in the flash spectrum show marked differences from the lines arising from neutral atoms and from the ionized states of other elements. They should be useful aids in important lines of astrophysical work.

*Laboratory research.*—The accurate determination of the wave-lengths of their spectral lines and the study of their relative intensities and appearance under varied conditions of excitation offer opportunities for laboratory investigations of much interest and importance.

*Presence in sun.*—Ten elements, in the ionized state, *La, Ce, Pr, Nd, Sa, Eu, Gd, Dy, Er,* and *Yb,* are shown as present with varying degrees of probability. For five, *Il, Tb, Ho, Tu,* and *Lu,* the data are inadequate either to indicate a fair probability of their presence in the sun or to eliminate them from further consideration.

The identification of Fraunhofer lines with those of elements recognized on the earth has a general interest because it tells what constituents are common to the earth and the sun, and whether any occur in one body and not in the other, results bearing on the important question of the uniformity of nature. For special classes of elements, identification has a further interest. In the case of the rare-earth elements it lies in the fact that these elements, having high atomic weights and not occurring in great abundance, would lie close to the photosphere and be important for the investigation of low levels in solar and stellar atmospheres.

The spectra of the rare-earth elements are rich in lines, but have relatively few of outstanding importance. Chance coincidences with solar lines will be frequent. Identifications based on coincidences alone therefore require accurately determined wave-lengths. Without measures to three decimal places, it is difficult to fix the tolerance in departure from real agreement. For elements indubitably recognized in the sun, such as *Fe* and *Ti,* for which accurate I.A. measures are available, the departures from exact coincidence with solar lines are positive for sun *minus* arc. For some five hundred lines of *Fe* the mean of $\lambda$ sun $-\lambda$ arc is $+0.0083$ A; for four hundred lines of *Ti* the mean is $+0.0075$ A.

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington,* No. 364.

In the case of the rare-earth elements, practically one-sixth of all possible elements, the requisite wave-length data are wanting except for *Ce*[1,2] and *La*.[2] There are recent two-place measures for *Nd*,[3] *Sa*,[3] *Gd*,[4] *Dy*,[4,5] and *Er*.[6] The average spacing of the Rowland lines is 0.2 A. For lines introduced at random, 30 per cent should by chance fall 0.03 A or less, from solar lines. In the case of elements for which no recent three-place tables are at hand, an element is assumed to be present if 50 per cent of the strong lines fall within the foregoing limit, provided that all, or all but very few, of the strongest have apparently corresponding solar lines. It is difficult, however, to hold to a hard-and-fast rule, and in some instances identification must rest upon a judgment based upon experience and a variety of considerations.

Although two-place tables of wave-lengths do not furnish adequate means for definitive identifications, collateral evidence is given by the intensities of the lines, by their behavior in the spectra of sun-spots and stars and in the spectrum of the flash. One soon learns from experience what relation of intensities between laboratory and solar lines is probable for any given element. The characteristic behavior of enhanced lines, their weakening in sun-spots, their strengthening in stars of the $\gamma$ Cygni type, and their prominence in flash spectra furnish for some elements confirmatory evidence of their presence in the sun's atmosphere.

According to Menzel,[7] the enhanced lines of the rare-earth elements show a very characteristic behavior on the moving-plate spectrum taken by Campbell at the eclipse of 1905. They are intense in the low-lying chromosphere, do not reverse suddenly at the limb to the absorption spectrum like normal neutral atoms, do not fade rapidly into the continuous background like $Ti^+$ and $Sc^+$, but still show in emission on the sun's disk, in some cases as far as 20″ within the limb.

For elements whose lines are assigned to multiplets, the identi-

[1] Klein, *Zeitschrift für wissenschaftliche Photographie*, **18**, 45, 1918.
[2] Kiess, *Scientific Papers of the Bureau of Standards*, **17**, 323 (No. 421), 1921.
[3] *Ibid.*, **18**, 201 (No. 442), 1923. [4] *Ibid.*, **18**, 695 (No. 466), 1923.
[5] Eder, *Sitzungsberichte der Akademie der Wissenschaften in Wien*, IIa, **127**, 1099, 1918.
[6] *Ibid.*, IIa, **125**, 383, 1916.
[7] *Publications of the Astronomical Society of the Pacific*, **39**, 359, 1927.

fication is greatly facilitated by knowing the relative probability of the occurrence of their lines. Among the spectra of the rare earths, only that of *La* has been analyzed, and it now seems that its system of electron orbits differs from that of the other so-called "rare earths."[1]

The presence of *Ce* and *La* in the sun is well established by the data in hand; both are ionized; both show the characteristic behavior of enhanced lines in spots, in the flash, and in $\gamma$ Cygni; and I.A. wave-lengths to three places are available for both.

Praseodymium was formerly included with neodymium under the name "didymium." The only three-place tables are in the Rowland system (1906). In the *Tabelle der Hauptlinien*[2] the wave-lengths are entered only to the second decimal, an indication of the present estimate of their accuracy. In a comparison $\lambda$ sun$-\lambda$ arc, 40 per cent of the strongest *Pr* lines fall within 0.03 A of solar lines of intensities $-2$ to $-1^3$, and about 10 per cent are masked by strong solar lines. On the basis of coincidences, the evidence of its presence in the sun is not very strong. Other considerations are the observations by King[4] that the lines arise from the ionized atom, as for *Ce* and *La*, and by Adams and Joy[5] that they probably occur in the spectrum of $\gamma$ Cygni. The lines in Table I are given in the *Revision of Rowland's Preliminary Table*,[6] soon to be published, as probably present in the sun. From the intensities of the corresponding solar lines it is evident that *Pr* alone does not produce solar lines stronger than $-1$. Although the evidence for its presence is strengthened by its probable occurrence in the flash and in $\gamma$ Cygni, the apparent absence of many of its important lines from the spectrum of the disk makes its presence still open to question. The intention has been to err on the conservative side when it is a question of a new

[1] H. D. Hubbard, *Periodic Chart of the Elements*, rev. ed., 1927–1928.

[2] Heinrich Kayser, Bonn, 1926.

[3] *Revision of Rowland's Table of Solar Spectrum Wave-Lengths;* oooo, ooo, oo, on the Rowland scale of intensities, are indicated by $-3$, $-2$, $-1$ in the *Revision*. See *Proceedings of the National Academy of Sciences*, **13**, 678, 1927.

[4] *Publications of the Astronomical Society of the Pacific*, **39**, 238, 1927.

[5] *Proceedings of the National Academy of Sciences*, **13**, 393, 1927.

[6] *Carnegie Institution of Washington, Publication*, No. 396; *Papers of the Mt. Wilson Observatory*, **3**, 1928.

identification. A more definitive judgment requires new laboratory measures in the I.A. system to three decimals.

Neodymium is represented in the solar spectrum by practically all of the very strong lines and by 55 per cent of the lines of moderate strength. The normal intensities of the solar lines are −1 to 0; lines stronger than 0 are either known or probable blends. They arise from the ionized atom (King), and when they occur in the

TABLE I

PRASEODYMIUM

| SUN | | Δλ SUN−ARC | KING ARC INT. | IDENTIFICATION IN SUN |
|---|---|---|---|---|
| λ I.A. | Int. | | | |
| 3877.200 | 1N | 0.00 A | 200 | −*Pr*+? |
| 3908.069 | −2 | + .02 | 150 | *Pr*+? |
| 3908.413 | 0 | − .02 | 200 | *Ce*+*Pr*+? |
| 3953.505 | 1 | .00 | 125 | −*Pr*+? |
| 3994.814 | −1 | .00 | 200 | *Pr*+? |
| 4008.738 | −1 | + .01 | 75 | *Pr*+? |
| 4044.845 | −2 | + .01 | 60 | *Pr*+? |
| 4056.561 | −1 | + .02 | 80 | *Pr*+? *Fe*? |
| 4143.162 | −2 | + .02 | 150 | *Pr*+? |
| 4172.289 | −2 | .00 | 50 | *Pr*+? |
| 4225.332 | −2 | − .01 | 150 | *Pr*+? *Sa*+ |
| 4297.753 | 0 | .00 | 80 | *Cr*−*Pr*+? |
| 4333.904 | −1 | − .01 | 100 | *Pr*+? |
| 4408.800 | −1N | − .03 | 200 | *Pr*+? |
| 4510.180 | −3 | + .03 | 100 | *Pr*+? |
| 4651.513 | −3N | − .01 | 75 | *Pr*+? |
| 5220.092 | −2 | − .01 | 50 | *Cu*?−*Pr*+? |
| 5322.817 | −2 | +0.04 | 60 | *Pr*+? |

spectrum of $\gamma$ Cygni it is with increased intensity; in the flash spectrum they have the characteristics of the rare-earth group. Table II gives the lines common to the sun and $\gamma$ Cygni.

The laboratory wave-lengths of *Nd* appear to be affected by systematically positive errors, which are taken into account in sun *minus* arc comparisons by allowing greater latitude for negative residuals.

For illinium the only data are the lines of unknown origin common to the spectra of *Nd* and *Sa*.[1] The strongest of these are listed in Table III, together with the corresponding solar lines. The absence of many of the strong lines indicated by the comparison with solar wave-lengths shows that the probability of its pres-

[1] Kiess, *Scientific Papers of the Bureau of Standards*, **18**, 218 (No. 442), 1923.

ence in the sun's spectrum is very small. A definitive conclusion must await the isolation of the pure element in amount sufficient for the measurement of its spectrum with adequate dispersion.

TABLE II

NEODYMIUM

| SUN | | Δλ SUN−ARC | γ CYGNI | | IDENTIFICATION IN SUN |
|---|---|---|---|---|---|
| λ I.A. | Int. | | λ I.A. | Int. | |
| 4135.297 | 0d? | −0.03 A | .35 | 3 | −*Nd*+ |
| 4232.388 | −1 | − .01 | .40 | 2 | *Nd*+ |
| 4284.538 | −1 | + .04 | .52 | 1 | *Nd*+? |
| 4314.514 | 0N | + .01 | .53 | 0 | *Nd*+ |
| 4338.696 | 0 | + .01 | .72 | 2 | −*Nd*+ |
| 4358.170 | 0 | − .03 | .14 | 1 | −*Nd*+ |
| 4385.677 | −1 | .00 | .66 | 2 | *Nd*+ |
| 4446.405 | −1 | +0.04 | .37 | 3 | *Nd*+ |

TABLE III

ILLINIUM

| SUN | | ARC | | Δλ SUN−ARC | IDENTIFICATION IN SUN |
|---|---|---|---|---|---|
| λ I.A. | Int. | λ I.A. | Int. | | |
| 5483.110 | 1 | 5483.10* | 3 | +0.01 A | *Fe* |
| 5508.420 | 0 | 5508.41* | 3 | + .01 | |
| ——— | | 5551.38 | 3 | | |
| ——— | | 5603.63* | 3 | | |
| ——— | | 5617.67* | 3 | | |
| 5698.894 | −3N | 5698.91* | 3 | − .02 | |
| ——— | | 5706.20† | 5 | | |
| 5739.988 | 0 | 5739.97* | 3 | + .02 | *Ti* |
| 5740.878 | −2 | 5740.86* | 5 | + .02 | Atm.? *Nd*+? |
| ——— | | 5829.70 | 3 | | |
| 5984.279 | −2 | 5984.32 | 3 | − .04 | |
| 5989.295 | 0 | 5989.33* | 3 | − .03 | Atm. wv. |
| ——— | | 6053.83 | 3 | | |
| 6084.118 | 0 | 6084.11 | 3 | +0.01 | |
| ——— | | 6803.06 | 3 | | |

* Enhanced line (King).

† Blend of *Sa* I and moderate *Nd* II (King).

Samarium is represented in the solar spectrum by 50 per cent of the strongest lines, of which but 25 per cent give sun *minus* arc differences greater than 0.025 A. The normal solar intensity of the strongest lines is probably −1. $Sa^+$ lines occur in the spectrum of γ Cygni with intensities much greater than in the solar

spectrum. Those common to the spectra of the sun and $\gamma$ Cygni are tabulated in Table IV. Some of these also appear in the flash spectrum with no other known identifications.

The weakening of $\lambda$ 4280 in $\gamma$ Cygni follows from the abnormal intensity of the solar line. The solar intensity is mainly attributable to an unidentified line which is weakened in, or absent from, $\gamma$ Cygni. Although $\lambda$ 4424 is the strongest $Sa^+$ line, its great strength in $\gamma$ Cygni is due mainly to a neighboring *Cr* line with which it is blended by the lower dispersion of the spectrum of $\gamma$ Cygni.

TABLE IV

SAMARIUM

| Sun | | $\Delta\lambda$ Sun−Arc | $\gamma$ Cygni | | Identification in Sun |
|---|---|---|---|---|---|
| $\lambda$ I.A. | Int. | | $\lambda$ I.A. | Int. | |
| 4256.423 | −1 | +0.02 A | .34 | 1 | *Sa*+ |
| 4262.713 | 0 | + .03 | .68 | 1 | *Sa*+− |
| 4280.790 | 1 | − .01 | .81 | 0 | *Sa*+− |
| 4318.940 | −1 | − .01 | .98 | 0 | *Sa*+ |
| 4329.053 | −1 | + .02 | .01 | 1 | *Sa*+ |
| 4334.195 | −1 | + .03 | .11 | 1 | *Sa*+? |
| 4421.131 | −1N | − .01 | .13 | 1 | −*Sa*+ |
| 4424.371 | −1 | + .02 | .33 | 4 | *Sa*+ |
| 4434.344 | −1 | .00 | .35 | 1 | *Sa*+− *Ti*? |
| 4452.740 | −1 | − .01 | .72 | 1 | *Sa*+ |
| 4467.337 | −1 | +0.01 | .36 | 1 | *Sa*+ |

Europium is probably represented in the solar spectrum by its strong enhanced lines. The two in the region covered by the plates of $\gamma$ Cygni appear to be present with greater relative intensity than in the solar spectrum. The strongest spark line, $\lambda$ 4129, is the only one that would not be confused with neighboring lines both in $\gamma$ Cygni and in the flash spectrum, and apparently it is present in both. The intensities of the corresponding solar lines, while fairly consistent with each other, are very strong in comparison with the lines of all the other rare-earth elements. This tends to weaken the evidence, but nevertheless the probability of the presence of $\lambda$ 4129 in the sun's spectrum is strong. Table V gives the data for *Eu*.

Gadolinium is an element for which accurate three-place measures are particularly necessary in determining whether it is present in the sun, and, if so, how widely its spectrum is represented. The

lines of intensities 4–6 fit solar lines as closely as those of intensities 10–20, and only accurate laboratory measures can lead to definite conclusions. It appears to be present in $\gamma$ Cygni, and S. A. Mitchell

TABLE V

EUROPIUM

| SUN | | ARC | SPARK INT. | $\Delta\lambda$ SUN–ARC | $\gamma$ CYGNI | | IDENTIFICATION | |
|---|---|---|---|---|---|---|---|---|
| $\lambda$ I.A. | Int. | $\lambda$ I.A. | | | $\lambda$ I.A. | Int. | Flash | Sun |
| 3819.690 | 1Nd? | .64 | 50R | +0.05 A | *v** | ..... | *He* | *Eu*+?– |
| 3930.515 | 0 | .51 | 50 | + .01 | *v* | ..... | *Fe* | *Eu*+? |
| 3971.996 | 0Nd? | .95 | 50 | + .05 | *v* | ..... | *Eu–Gd* | *Eu*+?– |
| 4129.732 | 1 | .72 | 100 | + .01 | .72 | 5 | *Eu* | *Eu*+ |
| 4205.029 | 1 | .03 | 100 | .00 | .06 | 8 | *V–Eu* | *Eu*+? |
| 4205.082 | 1 | .09 | ..... | −0.01 | | | | *V*+ |

* *v* = too far to violet.

TABLE VI

GADOLINIUM

| SUN | | $\Delta\lambda$ SUN–ARC | INTENSITY | | $\gamma$ CYGNI | | FLASH INT. | IDENTIFICATION IN SUN |
|---|---|---|---|---|---|---|---|---|
| $\lambda$ I.A. | Int. | | Arc | Spark | $\lambda$ I.A. | Int. | | |
| 3100.525 | −2 | +0.02 A | 8 | 8 | *v** | ..... | ..... | *Gd*+? |
| 3549.372 | 0 | .00 | 7 | 10 | *v* | ..... | 0 | *CN Gd*+? |
| 3646.197 | 1 | + .01 | 10 | 10 | *v* | ..... | 0d? | *Ti–Gd*+? |
| 3671.223 | −2 | − .02 | 10 | 8 | *v* | ..... | 6d | *V–Gd*+? |
| 3719.460 | −2N | − .02 | 9 | 10 | *v* | ..... | ..... | *Gd*+? |
| 3768.408 | 0 | + .01 | 10 | 10 | *v* | ..... | ..... | *Gd*+? |
| 4049.442 | −1 | .00 | 6 | 4 | *v* | ..... | 1 | *Mn Gd*+? |
| 4049.871 | −1 | − .03 | 8 | 6 | *v* | ..... | 0 | *Fe–Gd*+? |
| 4085.579 | −1 | − .01 | 8 | 8 | *v* | ..... | ..... | *Gd*+? |
| 4130.454 | 0 | + .06 | 10 | 10 | .42 | 0 | ..... | *Gd*+? |
| 4184.314 | 2 | + .03 | 9 | 10 | .34 | 5 | 3d | *Gd*+? *Ti*+ |
| 4251.751 | −1 | .00 | 8 | 10 | .72 | 1 | 2 | *Gd*+?–*Ti* |
| 4262.136 | −1 | + .05 | 9 | 10 | 1.93 | 8 | ..... | *Cr*+*Gd*+? |
| 4327.159 | −1 | + .05 | 8 | 4 | ..... | ..... | 1 | *Gd*+? |
| 4342.192 | 0 | +0.01 | 10 | 10 | .13 | 2 | 1 | *Gd*+? *V* |

* *v* = too far to violet.

identifies twenty-two solar lines in the flash spectrum as *Gd*, but of these, seventeen are blends. The most favorable showing for its presence in the sun is given in Table VI; this suggests its occurrence in the ionized state, though only six of the lines have no other probable identification.

No terbium lines were found by Adams and Joy in the spectrum

of $\gamma$ Cygni, or by Mitchell in the flash spectrum, but the presence of $Tb^+$ is considered by Menzel[1] as provisionally probable in the flash spectrum taken by Campbell in 1905 with a moving-plate camera. Thirty-seven solar lines were identified by Hofbauer as due to *Tb*, but from the results given in Table VII it is evident that the available laboratory data are not sufficiently accurate to show its presence in the sun through coincidences with solar lines. The weak lines agree with solar lines as closely as the strongest lines. The data for the strongest spark lines are given below, and, while failing to show any high probability of its presence in the sun, they do show how urgently new determinations of wave-lengths are needed.

TABLE VII

TERBIUM

| SUN | | EBERHARD | | EXNER AND HASCHEK | | | | $\Delta\lambda$ SUN − MEAN $\lambda$ | IDENTIFICATION IN SUN |
|---|---|---|---|---|---|---|---|---|---|
| | | | | Arc | | Spark | | | |
| $\lambda$ I.A. | Int. | $\lambda$ I.A. | Int. | $\lambda$ I.A. | Int. | $\lambda$ I.A. | Int. | | |
| 3702.826... | −2Nd? | .87 | 8 | .87 | 8 | .86 | 15 | −0.04 A | ..... |
| 3776.561... | 1 | .48 | 7 | .52 | 8 | .51 | 8 | + .06 | *Mn V+* |
| 3848.708... | −1 | .78 | 4 | .77 | 20 | .77 | 20 | − .06 | *CN* |
| 3874.125... | −1 | .17 | 6 | .20 | 20 | .19 | 20 | − .06 | *CN* |
| 3899.144... | 2 | .20 | 8 | .20 | 10 | .18 | 8 | − .05 | *V+* |
| 3939.520... | −1 | .54 | 8 | .61 | 15 | .57 | 10 | − .05 | ..... |
| 3976.870... | 2 | .85 | 10 | .87 | 20 | .86 | 10 | + .01 | *Fe Nd+* |
| 3982.002... | 2 | .88 | 10 | .93 | 15 | .90 | 10 | + .10 | *Ti+ Fe?* |
| 4005.484... | 1 | .48 | 8 | .56 | 15 | .55 | 10 | − .05 | *Fe* |
| 4278.553... | −3 | .56 | 10 | .55 | 10 | .52 | 10 | +0.01 | ..... |

For dysprosium there are two-place I.A. measures (1918). Strong lines for which there are no other identifications show fair agreement with solar lines of intensity −1. The element is not identified in $\gamma$ Cygni, but Mitchell gives three lines, which, however, are accounted for by known constituents of the sun's atmosphere; and Menzel[2] finds five lines which show the behavior typical of rare earths and which are not otherwise accounted for. These are indicated by an asterisk in Table VIII. Although the spark intensities, on the whole, are less than the arc intensities, the element

[1] *Publications of the Astronomical Society of the Pacific*, **39**, 359, 1927.

[2] Communicated by letter.

is considered to be in the ionized state because of its similarity to the other rare earths in its behavior in the flash spectrum.

For holmium there is little spectroscopic evidence in the sun. When the data in the *Tabelle der Hauptlinien* are used for the comparison, one-third of the strong lines fall within 0.03 A of solar lines. Many are apparently absent. A few are identified by Mitchell and Baxandall[1] as *Ho* in the sun, but though they are in the region covered by Campbell's moving-plate spectrum, none of them is reported by Menzel. The solar lines identified as *Ho* by Mitchell or

TABLE VIII

DYSPROSIUM

| SUN | | ARC λ I.A. | INTENSITY | | Δλ SUN−ARC | IDENTIFICATION IN SUN |
|---|---|---|---|---|---|---|
| λ I.A. | Int. | | Arc | Spark | | |
| 3531.710 | 1 | .70 | 10 | 10 | +0.01 A | *Dy*+?− |
| 3544.230 | 1 | .24 | 8 | 3 | − .01 | −*Dy*+? |
| 3550.223 | 1N | .21 | 8 | 10 | + .01 | *CN Dy*+? |
| 3944.686* | 1 | .69 | 10 | 10 | .00 | *Fe*?−*Dy*+ |
| 4045.970 | 1N | .00 | 10 | 4 | − .03 | −*Dy*+? |
| 4077.977 | 0 | .98 | 10 | 10 | .00 | *Dy*+? |
| 4103.319* | 0 | .34 | 8 | ..... | − .02 | −*Dy*+ |
| 4186.799 | −3 | .80 | 8 | 4 | .00 | *Dy*+? |
| 4194.851* | 1 | .85 | 8 | 4 | .00 | *Dy*+− |
| 4256.317* | −1 | .33 | 8 | 3 | − .01 | *Dy*+ |
| 4449.721* | −1 | .72 | 8 | 4 | .00 | *Dy*+ |
| 4612.272 | −2 | .27 | 8 | 4 | 0.00 | *Dy*+? |

* Shows characteristic rare-earth behavior in flash spectrum.

Baxandall are shown in Table IX together with the identifications in the *Revised Rowland Table.*

Identifications of solar lines as due to erbium have been suggested by Mitchell, Baxandall, and Menzel. The complete data are given in Table X. The first and second columns give the solar wavelengths in I.A. and the Rowland intensities. The data in the next three columns are mainly from the *Tabelle der Hauptlinien.* Then follow findings by King from an examination of arc, spark, and furnace spectra, and the sun *minus* arc differences. The suggestions by Baxandall, Mitchell, and Menzel are given under their respective names, while the solar identifications appear in the last column.

[1] A revision of identifications has been generously placed at our disposal by F. E. Baxandall.

The rare-earth elements known to occur in the sun are from the ionized atom. The evidence for *Er* is given by the two strongest ionized lines, λ 3896 and λ 3906. For λ 3896 the evidence is the strongest, but for λ 3906 it is less definite. This is the strongest ionized line and appears in the flash with the earth-element character, combined with that of a strong normal line of *Co*, but the solar line is much too strong for *Er* alone. It is evident that the normal solar intensity of $Er^+$ does not exceed −1. Identification of solar lines with weak enhanced lines is at best very questionable. Only two of them, λ 4008 and λ 4251, give indication of their

TABLE IX

HOLMIUM

| SUN | | "TABELLE DER HAUPTLINIEN" | | | Δλ SUN−ARC | IDENTIFICATION | | |
|---|---|---|---|---|---|---|---|---|
| λ I.A. | Int. | λ I.A. | Arc Int. | Spark Int. | | Mitchell | Baxandall | Sun |
| 3456.015 | −1 | .00 | 10 | 10 | +0.02 A | *Er−Ho* | ..... | ..... |
| 4045.970 | 1N | .... | .... | .... | ..... | ..... | *Ho?* | *−Dy+?* |
| 4053.938 | −2 | .92 | 10 | 8 | + .02 | ..... | *Co, Ho?* | *Co?* |
| 4065.091 | 2d? | .... | .... | .... | ..... | *V−Ti* | *Mn Ti V+ Ho?* | *Ti Mn* |
| 4108.536 | 2 | .60 | 10 | 5 | − .06 | *Ho, Er* | *Ca?* | *−Ca?* |
| 4350.762 | −3 | .73 | 10 | 5 | +0.03 | ..... | *Ho?* | ..... |

presence in the flash spectrum. For the very weak lines the evidence is nil, except for λ 4081, a possible blend with *Fe*, and a suspicion of λ 4552. Lines from the normal atom would not be expected to appear in the solar spectrum, and there is no dependable indication that they do. Erbium illustrates the uncertainties inherent in identification by the method of coincidences in the absence of adequate wave-length data.

The evidence for thulium in the sun is very weak. If present, the normal solar intensity of the strongest lines would be about −2. The sun *minus* arc differences cover so wide a range that laboratory errors of the order of 0.1 A must be assumed to reconcile the discrepancies. Here again, more accurate wave-lengths are needed to show its presence in the sun. The comparison of *Tu* with solar wave-lengths is given in Table XI.

Ytterbium, called also neo-ytterbium and aldebaranium, has,

## TABLE X

### ERBIUM

| SUN | | LABORATORY | | | KING | | $\Delta\lambda$ SUN − ARC | IDENTIFICATION | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| λ I.A. | Int. | λ I.A. | Int. Arc | Int. Sp. | | Int. Sp. | | Baxandall | Mitchell | Menzel | Sun |
| 3896.250 | −1 | .26 | 6 | 6 | *Er*+ | 10 | −0.01 A | | *Er*−*V* | c* | *Er*+? |
| 3906.302 | 2 | .34 | 10 | 10 | *Er*+ | 20 | − .04 | *Co, Ni, Er* | | d† c | *Co Er*+? |
| 3938.632 | 0 | .65 | 8 | 4 | *Er*+ | 3 | − .02 | *Er* | | | |
| 3973.124 | 1 | .26 | 4 | | *Er* | | − .11 | *Er* | | | *Co* |
| 3973.170 | 1 | | | | | | | | | | |
| 3973.658 | 1 | .61 | 4 | 2 | *Er* | | + .05 | *Fe, V*+, *Nd, Er* | | | *Fe V*+ −*Nd*+ |
| 3974.766 | 6d? | .72 | 5 | 5 | *Er*+ | 1 | + .05 | *Co, Fe, Er* | *Co*−*Er*−*Ni* | | *Fe Co* |
| 4007.931 | 0N | .96 | 10 | 4 | *Er* | | − .03 | | | c | *Co?* |
| 4008.178 | 0 | .22 | 3 | | *Er*+ | <1 | − .04 | *V Er* | *Ti*−*Er* | | *V*+? |
| 4012.611 | −1d? | .57 | 4 | 2 | *Er* | | + .04 | *Er* | | | |
| 4015.613 | 3 | .63 | 5 | 3 | *Er*+ | 1 | − .02 | *Ni*+, *Er* | *Ni*−*Er*−*La* | m‡ | |
| 4077.977 | 0 | | | | *Dy*+ | | | *Er, Dy* | | | *Dy*+? |
| 4081.236 | 0 | .3 | 7 | 4 | *Er*+ | 3 | .0 | *Zr, Ce, Ti* | *Zr*−*Ce*−*Er* | c | *Ce*+ *Zr* |
| 4081.266 | 1 | | | | | | | *Er, Fe?* | | | *Fe*− |
| 4105.830 | −2 | .85 | 3 | | *Er* | | − .02 | *Er, Tm* | | | |
| 4140.164 | −2 | .20 | 1 | | *Er*+ | <1 | − .04 | *Er, Mn* | | | |
| 4187.592 | 2 | .62 | 3 | | *Er* | | − .03 | *Fe, Er* | *Fe*−*Ti*−*Ni* | | *Fe* |
| 4189.992 | 0N | .99 | 5 | | *Er*+ | <1 | .00 | *Mn, Er* | *Er*−*Mn*−*C* | | *Mn* |
| 4190.719 | 1Nd? | .72 | 4 | 3 | *Er* | | .00 | *Co, Cr, Er, C* | *Co*−*Er*−*C* | | *Co CN* |
| 4194.851 | 1 | .81 | 10R | 2 | *Er* | | +0.04 | *Co, Dy* | | c | *Dy*+− |

* Shows characteristic behavior of rare-earth lines.

† Normal neutral line showing reversal.

‡ Masked.

TABLE X—*Continued*

| Sun | | Laboratory | | | King | | | Identification | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| λ I.A. | Int. | λ I.A. | Int. Arc | Int. Sp. | | Int. Sp. | Δλ Sun−Arc | Baxandall | Mitchell | Menzel | Sun |
| 4218.399 | 1Nd | .43 | 4 | 3 | *Er* | ..... | −0.03 | *Er* | *Zr, Er* | ..... | ..... |
| 4242.164 | 0 | .11 | 4 | ..... | *Er*+ | 2 | + .05 | *Er* | ..... | ..... | ..... |
| 4251.889 | −2 | .92 | 5 | 5 | *Er*+ | 2 | − .03 | ..... | ..... | c d | ..... |
| 4286.589 | −1 | .55 | 5 | 2 | *Er* | ..... | + .04 | *Er* | ..... | ..... | ..... |
| 4303.836 | 2 | .80 | 4 | 2 | *Er*+ | <1 | + .04 | *Er* | *Nd Er* | ..... | ..... |
| 4319.981 | −1N | .94 | 5 | ..... | *Er*+ | 1 | + .04 | *Er* | ..... | ..... | ..... |
| 4374.946 | 2 | .95 | 10 | ..... | *Y*+ | ..... | .00 | *Mn, Co, Nd* | ..... | m c? | *Mn Co−Y*+ |
| 4384.714 | 3 | .72 | 5 | 3 | *Er*+ | 1 | − .01 | *V, Er* | *V* | ..... | *V* |
| 4386.460 | 0 | .40 | 4 | 2 | *Er* | ..... | + .06 | *Ni, Er* | ..... | ..... | *Ni* |
| 4409.369 | −1N | .34 | 5 | 2 | *Er* | ..... | + .03 | *Er, Dy* | *Ti−Er* | ..... | *Sa*+ |
| 4419.609 | −2 | .62 | 8 | 10 | *Er*+ | 5 | − .01 | *Er* | *Er−Pr* | abs. | ..... |
| 4500.769 | −2 | .75 | 8 | 6 | *Er*+ | 2 | + .02 | *Er* | ..... | ..... | ..... |
| 4552.145 | 0 | .12 | 6 | 4 | *Er*<br>*Er*+ | .....<br>2 | + .03 | *Er* | ..... | abs. | ..... |
| 4675.614 | 0 | .61 | 5 | 10 | *Er*+ | 3 | .00 | *Er* | ..... | ..... | *Ni* ? |
| 4724.540 | −2 | .54 | 6 | 3 | § | ..... | .00 | *Er* | ..... | ..... | ..... |
| 4759.673 | −3 | .67 | 5 | 3 | § | ..... | .00 | *Er* | ..... | ..... | ..... |
| 4762.633 | 1 | .65 | 6 | 3 | § | ..... | −0.02 | *Ni, Er* | *Mn−Er* | ..... | *Ni* |

§ Beyond range of plate.

according to Exner and Haschek, three outstanding lines. Two of them, λ 3289 and λ 3694, agree in position with solar lines of intensities 4 and 3. The third, λ 3987, is the strongest, and is 0.04 A to the red of a solar line of intensity 0. The solar line of intensity 4 is identified as $V^+$; the lines of intensities 3 and 0 have no other identifications. Although all three fall near solar lines and were suggested in the Appendix of the *Preliminary Table*, only two have been retained as possible *Yb* lines. Menzel states that λ 3987 is definitely absent from the flash spectrum and that the region is one where the dispersion is high and the definition good on Campbell's plates. The intensities by Exner and Haschek also indicate

TABLE XI

THULIUM

| SUN | | "TABELLE DER HAUPTLINIEN" | | | Δλ SUN—ARC | IDENTIFICATION IN SUN |
|---|---|---|---|---|---|---|
| λ I.A. | Int. | λ I.A. | Arc. Int. | Spark Int. | | |
| 3133.967 | −1Nd? | .87 | 9 | 9 | +0.10 A | ..... |
| 3425.064 | −2 | .12 | 7 | 7 | − .06 | *V* |
| 3441.553 | −3N | .53 | 7 | 6 | + .02 | ..... |
| 3701.376 | −2 | .37 | 9 | 9 | + .01 | ..... |
| 4094.071 | −2 | .18 | 10R | 5 | − .11 | ..... |
| 4242.164 | 0 | .15 | 10 | 8 | + .01 | ..... |
| 4359.909 | −1 | .93 | 8 | 5 | −0.02 | ..... |

that this is an arc line. That *Yb* contributes to λ 3694, a line of intensity 3, is indicated by the relatively high velocity of outflow from spots for a line of this intensity, a characteristic of low level. Its probable level is somewhat below that of *Ce* and *La*.[1] The comparison of *Yb* lines with solar lines is given in Table XII.

As to lutecium, also called cassiopeium, the presence of its lines in the solar spectrum is very doubtful. The probability for or against its presence requires for its determination more precise measures of wave-length. The five lines of intensity 10 in the arc and spark given in the *Tabelle der Hauptlinien* are listed in Table XIII. As three of these are probably masked in the solar spectrum and the other two give large residuals, this element has not been added as present in the sun.

[1] *Mt. Wilson Contr.*, No. 74; *Astrophysical Journal*, **38**, 345, 1913.

The results from the discussion are given in Table XIV. Ten elements, $La^+$, $Ce^+$, $Pr^+$, $Nd^+$, $Sa^+$, $Eu^+$, $Gd^+$, $Dy^+$, $Er^+$, and $Yb^+$, are shown as present in the sun with varying degrees of probability. For five, *Il*, *Tb*, *Ho*, *Tu*, and *Lu*, the data are not adequate either to establish the probability of their presence or to eliminate them from further consideration.

TABLE XII

YTTERBIUM

| SUN | | "TABELLE DER HAUPTLINIEN" | | | EXNER AND HASCHEK | | Δλ SUN−ARC | IDENTIFICATION IN SUN |
|---|---|---|---|---|---|---|---|---|
| λ I.A. | Int. | λ I.A. | Arc Int. | Spark Int. | Arc Int. | Spark Int. | | |
| 3107.855 | −1 | .87 | 10 | 10R | 3 | 5r | −0.02 A | ..... |
| 3289.373 | 4 | .37 | 10R | 10 | 100R | 100 | .00 | *Yb*+? *V*+ |
| 3343.025 | −1 | .96 | 10 | 10 | 4 | 3 | + .06 | *NH*− |
| 3464.475 | 1 | .33 | 10 | 5 | 3r | 3 | + .14 | *Sr*+ |
| 3478.786 | 0 | .84 | 8 | 10 | 5 | 20 | − .06 | *Fe* |
| 3694.200 | 3 | .20 | 10R | 10 | 200R | 200 | .00 | *Yb*+?− |
| 3987.972 | 0 | .01 | 10 | 10 | 500R | 15 | − .04 | ..... |
| 4439.171 | 0Nd? | .22 | 8 | 2 | 10 | 2 | − .05 | ..... |
| 5556.476 | −3 | .47 | 10 | 1 | ..... | ..... | +0.01 | *Y* |

TABLE XIII

LUTECIUM

| SUN | | ARC λ I.A. | Δλ SUN−ARC | IDENTIFICATION IN SUN |
|---|---|---|---|---|
| λ I.A. | Int. | | | |
| 3397.063 | −3 | .02 | +0.04 A | ..... |
| 3507.405 | 1 | .40 | + .01 | *Fe* |
| 3554.453 | 2 | .43 | + .02 | ..... |
| 4184.314 | 2 | .24 | + .07 | *Gd*+? *Ti*+ |
| 5476.745 | −3 | .70 | +0.05 | ..... |

The rare-earth elements are of much astrophysical importance. Their vapors lie close to the photosphere and are apparently easily ionized. The low level of $La^+$ and $Ce^+$ is shown by high velocity of outflow from spots and by the low values they give for solar rotation. In the sun and in $\gamma$ Cygni the $Ce^+$ lines show large displacements to the violet, relative to the displacements of medium-level lines in the same spectral region, a characteristic of low-level lines.

Accurate laboratory data would decide whether the still doubtful elements are present or absent from the portions of the sun accessible to spectroscopic observation, and, if present, how widely their spectra are represented. For other purposes also, the wave-lengths of the rare-earth elements need to be known with high precision. Their determination offers a promising field of investigation, with sufficient uncertainties, possibilities, and questions of technique to be interesting.

TABLE XIV

RARE-EARTH ELEMENTS

| Atomic Number | Symbol | Element | Identification in Sun | | Available Wave-Length Data | | | |
|---|---|---|---|---|---|---|---|---|
| | | | Present | Max. Int. | Measures | System | Source | Date |
| 57 | *La* | Lanthanum | *La*+ | 1 | 3-place | I.A.* | Bur. St. | 1921 |
| | | | | | 3-place | Rowl. | Wolff | 1905 |
| 58 | *Ce* | Cerium | *Ce*+ | 0 | 3-place | I.A. | Klein | 1918 |
| | | | | | 3-place | I.A.* | Bur. St. | 1921 |
| 59 | *Pr* | Praseodymium | *Pr*+ ? | −1 | 3-place | Rowl. | Bertram | 1906 |
| 60 | *Nd* | Neodymium | *Nd*+ | 0 | 3-place | Rowl. | Bertram | 1906 |
| 61 | *Il* | Illinium†‡ | ..... | ..... | 2-place | I.A.* | Bur. St. | 1922 |
| 62 | *Sa* | Samarium | *Sa*+ | −1 | 3-place | Rowl. | Rütten and Morsch | 1905 |
| 63 | *Eu* | Europium | *Eu*+ ? | 1? | 2-place | I.A. | Eder | 1917 |
| 64 | *Gd* | Gadolinium | *Gd*+ ? | 0 | 2-place | I.A.* | Bur. St. | 1923 |
| 65 | *Tb* | Terbium‡ | ..... | ..... | 2-place | Rowl. | Exner and Haschek | 1911–1912 |
| 66 | *Dy* | Dysprosium | *Dy*+ | 0 | 2-place | I.A.* | Bur. St. | 1923 |
| 67 | *Ho* | Holmium‡ | ..... | ..... | 2-place | Rowl. | Eder and Valenta | 1910 |
| 68 | *Er* | Erbium | *Er*+ ? | −1 | 2-place | I.A. | Eder | 1916 |
| 69 | *Tu* | Thulium‡ | ..... | ..... | 2-place | Rowl. | Exner and Haschek | 1911–1912 |
| 70 | *Yb* | Ytterbium | *Yb*+ ? | −1? | 3-place | Rowl. | Kayser | 1903 |
| 71 | *Lu* | Lutecium‡ | ..... | ..... | 3-place | Rowl. | Kayser | 1903 |

* λ 5500 to red.

† For *Il* the lines common to the spectra of *Nd* and *Sa* were used in Table III.

‡ Available wave-length data are not sufficiently accurate for the determination of either its presence or its absence with a reasonable degree of probability.

Numerous solar lines of intensities −3 to −1 are still unidentified, and it is probable, or at least possible, that many of them are due to the rare-earth elements. The question of their presence in the sun could be decided by accurate wave-lengths of the stronger enhanced lines; the extent to which they are represented, by a study of lines of decreasing strength in laboratory sources. A third question is the state of ionization, and whether both the normal and ionized states are represented.

The lines of the rare earths appear unique in their behavior in

the flash spectrum, and for enhanced lines they show a peculiar behavior in the arc, where they are often stronger than in the spark. The enhanced lines of *Ti* and *Sc* are prominent at high levels in the sun's atmosphere, while those of the rare-earth elements are confined to lower levels. In the first case, high ionization has been attributed to lower pressure at high levels; in the second case, excitation by high temperature near the photosphere may play a larger rôle. A promising field of investigation would seem to be a study of the effects of varying the conditions under which the spectra are produced in the laboratory.

It is evident from the preceding discussion that complete agreement does not obtain, nor can it be expected in the identifications given by different investigators. The best one can do in the cases where definitive conclusions are not conservatively possible, is to suggest what seems fairly probable and leave the final decision to later consideration in the light of further observation.

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